The Economics of Traffic Congestion: A Quantitative Analysis

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Dedication

To my family and friends for standing by me over all this process.

Thanks!

Abstract

In this dissertation I explore the effect that traffic congestion has in shaping commuting choices and the distribution of economic activity and people in cities.

In the first two chapters I set up a framework to evaluate the welfare impact of investments in the transportation network in a city. Chapter 1 theoretically lays out this framework and chapter 2 applies it to evaluate first, the extension of the Expo rail line in Los Angeles county and second, the value of the entire rail network in Los Angeles county.

In the first chapter (1) I develop a structural model of the commuting market in a city. A city is a collection of residential and working locations connected by a transportation network. The transportation network is composed of the road network and the public transit and street network. The crucial difference between the road network and the other two is that the first can be congested as a function of usage. Moreover, there is an exogenous demand for travel from residential to employment locations. On the demand side, commuters make mode and routing choices taking as given trip characteristics (travel time and travel money cost). On the supply side, the transportation network maps travel demand into trip characteristics. An equilibrium is reached when, given trip characteristics, commuting mode shares give rise to these trip characteristics and vice versa.

To solve this problem I use methods from the civil engineering literature on traffic assignment. There is a plethora of algorithms that allow to solve the routing problem in a congested road network efficiently. I outline an algorithm that allows to solve for the commuting market equilibrium that has nested the traffic assignment problem.

The second chapter (2) brings the model developed in the first part to the data and use it to evaluate the effect of the Expo rail line extension and the value of the entire rail network in Los Angeles counties. I start by estimating demand side parameters

using a mix of the California Household Travel Survey augmented by requests to Google maps. With parameter estimates I find that the value of time is $19.81 per hour which is in line with the median hourly wage in Los Angeles of $20.52 per hour. Next, I estimate the parameters of the congestion function using highly disaggregated highway flow and speed data from the California Department of Transportation. My parameter estimates differ from the ones suggested by the Bureau of Public Roads indicating that the Los Angeles county highway system gets congested for lower car volumes than previously thought. Then I assess the accuracy of the model and find that the model capture accurately the main moments of the data: mode shares and average travel times.

In the first counterfactual I evaluate the effect of the Expo rail line extension. This extension connects Santa Monica to Los Angeles and had a cost of $1.5 billion. I find that this extension increased public transit share by 0.68 percentage points and reduced total travel time by 1,472 hours each 30 minutes. Welfare increased by 0.085% in the county and it takes 6.37 years to recover the investment. In the absence of congestion externalities the effect of the extension is more than double.

In the second countefactual I find that the value of the entire rail system in Los Angeles county is of $1.9 billion. This is in line with other studies that estimate congestion relief of the public transit system in Los Angeles county.

The last chapter of this dissertation (3) investigates the relationship between population density and commuting choices motivated by the fact that cities across the United States are considering changes in single-family zoning. The elimination of single-family zoning restrictions will make urban areas more dense and my conjecture is that this will impact the commuting market of these cities.

First, I study the relationship between population density and commuting mode choice by means of a reduced form exercise. I find that a 20% increase in population density is correlated with a decrease between 0.7 and 0.95 percentage points in private car commuting mode. Moreover I find that this decrease in private car mode share is captured almost entirely by the public transit system.

Next, I develop a model of internal city structure with residential and productive amenities, a fixed measure of development land, and endogenous travel costs with commuting mode choices. In short, people want to locate in areas with high amenity levels (agglomeration forces) but at the same time land prices and car travel times increase (dispersion forces). In this model, a change in zoning regulation is equivalent to increasing the amount of urban development land per location. Therefore, this model is able to simulate different zoning policies and capture the relationship between population density and commuting mode choices.

Finally, in the last section of this chapter I use the model to simulate how different uniform increases in population density across all locations affect mode choice and travel times in Los Angeles county. I find that a 20% increase in population density is associated with a decrease of 0.82 percentage points in car commuting share and that 90.3% of this decrease is captured by the public transit system. Therefore, the model is able to reproduce the empirical relationship previously documented. However, this increase in density is keeping all else constant, in particular where people live and work. Zoning policy changes that liberate residential land will not have a uniform effect across other locations and the full model will be to put to work to capture this fact. However, this is left as future work.

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Chapter 1

A Commuting Market Model With Endogenous Traffic Congestion Externalities

1.1 Introduction

Economic activity and population are highly concentrated in big cities. More than 80% of Americans live in urban areas. On the one hand, agglomeration have obvious benefits such as productivity and residential spillovers (Ahlfeldt et. al. (2015) ([**?**])) but on the other hand, this vast concentration of economic activity involves the transport of millions of people each day between their residence and workplace leading to congestion externalities. Traffic congestion is a major cost in big cities: the average commuter spends 54 hours per year in traffic delays and the total cost for the US economy is about $88 billion when factoring in time lost, pollution, and accidents.[1] To alleviate these costs local governments make big investments in transportation infrastructure. In the 2016 national election, cities and counties around the US decided on more than $200 billion in transit funding[2].

[1]from Texas A&M Transportation Institute and INRIX 2019 Index

[2]See this article for a list of the major projects passed.

Given the budget size of such projects it is crucial to have a framework that accurately estimates their impact. The expansion of the public transportation network will attract commuters (lower public transit travel times and new connections) and reduce the usage of the road network. If congestion is not considered, travel times on the road network are fixed and commuters do not impose any cost on all other commuters they encounter on the road. However, in the presence of congestion externalities, travel times depend on the network usage. If less commuters use the network, travel times will decrease making car commuting more attractive. Therefore, when evaluating the impact of changes in the transportation infrastructure in a city the model needs to accurately predict how travel times and commuter mode and routing choices will be affected. This presents the challenge of internalizing the commuting choice (mode and route) of millions of people in a spatially disaggregated framework that allows to evaluate the general equilibrium impacts of transportation investments.

Actual spatially disaggregated models abstract from traffic congestion effects. This is a simplifying but unrealistic assumption when considering highly populated urban areas. Figure (1.1) shows average 5 minute average flow of cars (blue line, left vertical axis) and speed (red line, right vertical axis) during a typical weekday in the freeway system of Los Angeles county. During night time, average speed is at free flow speed level, that is, the speed level at which congestion has not kicked in, around 65 miles/hour. As flow of cars increases, speed starts to decrease and stays under the free flow speed level from 6am to 10pm, even during off peak hours. That is, the freeway system is congested during 16 hours during a weekday.

In this paper I focus on the effect of improvements in the transportation network when commuting flows are given. To do so I develop a spatial structural model of the commuting market of a city. Commuters, given trip characteristics (travel time and travel cost), make commuting mode and routing choices. These choices map into trip characteristics that depend on the transportation network usage. This triggers a feedback loop between commuter choices and trip characteristics generating traffic

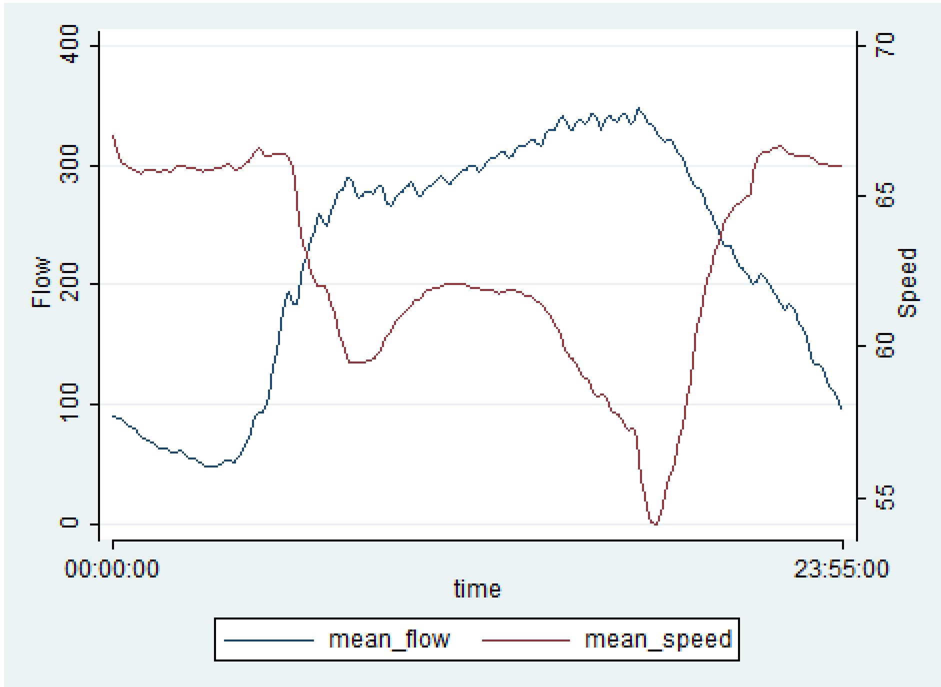


Figure 1.1: Congestion in Los Angeles

congestion externalities in the whole network. Hence, commuter choices in one side of the city can affect commuter choices in the other side because trip characteristics are affected through the externalities they impose in all other commuters.

The main contribution of the paper is to internalize trip characteristics as a function of commuter choices in a parsimonious way that allows to compute the equilibrium of a spatially disaggregated model. To do so I take advantage of civil engineering tools that allow to solve the city routing problem efficiently. For a given origin-destination matrix of car travel demand, the routing problem in the road network is known in the civil engineering literature as traffic assignment. The solution to this problem is based in the Nash Equilibrium concept: no commuter can find a profitable (routing) deviation given the routing choices of all other commuters. The numerical solution of this problem is based in the Frank-Wolfe convex combination algorithm (1956) which was adapted to the traffic assignment problem by Florian (1976) ([**?**]).

The model differs from others in the fact that travel characteristics arise endogenously as a function of commuters choices. Another particularity of the model is that, given congestion externalities, an improvement of the transportation network doesn't have to be welfare improving. Commuters don't internalize the cost their choices impose over all other commuters. However, in models without congestion externalities, any improvement that reduces travel times is necessarily welfare improving.

Next, I bring the model to the data to replicate the Los Angeles County transportation market and evaluate different counterfactuals. First, I estimate demand and supply side parameters. I use the 2012 California Household Travel Survey (CHTS-2012) to obtain data on realized mode choice and trip characteristics. However, to estimate demand I need to know trip characteristics of non realized commuting mode choices. I use start location, end location, day of the week, and start time of realized trips observed in the CHTS-2012 to obtain counterfactual trip characteristics data of non realized mode choices from Google Maps. Then, I estimate the demand model and find an estimated Value of Time of around \$20 per hour, which is in line with the median hourly wage in 2018 (\$20.52)[3].

On the supply side I use rich spatially disaggregated measures of cars flow and average speed over the whole freeway network in Los Angeles county from the Caltrans Performance Measurement System to estimate parameters governing the congestion effects. First I show that the car flow-speed relationship in the model is observed against car density in the data. Then I follow Kucharski and Drabicki (2017) to provide, what are to the best of my knowledge, the first estimates of congestion parameters in Los Angeles county. I find that the Los Angeles county freeway system suffers from congestion for lower levels of traffic flow than previously thought.

Then I use the model to analyze the extension of the Expo rail line in Los Angeles county. This line allows the connection between Santa Monica and Los Angeles

[3] Bureau of Labor Statistics

downtown by means of 6.6 rail miles and 7 new stations. It was open to the public in May 20th 2016 and had a cost of $1.5 billion. I find that this extension increased public transit share by 0.68 points and reduced total time traveled by 1,472 hours (0.24%) every 30 minutes. Welfare increased by 0.085%. With my estimates I find that the time to recover the investment is 6.37 years. However, when congestion is not included in the model, I find that total time reduction is 3,406 hours (0.57%) per 30 minutes, the welfare impact is 0.198% and time to recover the investment is 2.67 years. Congestion is erasing part of the welfare gains due to routing and commuting choices triggered by the extension.

In the second counterfactual I evalue the impact of the whole light rail network in Los Angeles county. To do so I simulate the model in the absence of all the light rail network and compare it to the equilibrium of the base scenario. I find that the value of the network is, at least, $1.9 billion per year. This result is in line with the finding in Anderson (2013) ([?]), who exploited the strike by Los Angeles County Metropolitan Transportation Authority workers to conclude that the cost relief of operating the public transit system (light rail and bus) is in the range of $1.2 to $4.1 billion per year.

Related Literature

First of all, this paper contributes to the literature on the economics of traffic congestion. On the one hand, a theoretical strand initialized by Vickrey (1963) ([?]) and continued by Arnott, de Palma and Lindsey (1993) ([?]) that makes simplifying assumptions such as a linear city. My model allows for a realistic network with heterogeneous locations where congestion arises endogenously and disperses through the network by changing commuting patterns of travellers. On the other hand, empirical papers such as Duranton and Turner (2011) ([?]), Couture, Duranton and Turner (2016) ([?]) and Akbar and Duranton (2017) ([?]) use a reduced form approach to estimate the cost of congestion whereas I use a structural model that allows to undertake counterfactual exercises. Moreover, this paper is closely related to Anderson

(2014) ([?]) who uses the sudden strike in 2003 by Los Angeles transit workers to use a reduce form approach to estimate the congestion relief benefit of the public transit system between $1.2 and $4.1 billion. However, I use a structural approach to find that the value of the rail transit system is $1.9 billion.

The paper also contributes to the emerging literature evaluating the impact of commuting and transportation infrastructure investments in a general equilibrium framework. Amongst these papers Ahlfeldt, Redding and Sturm (2016) ([?]), Allen and Arkolakis (2016) ([?]), and Tsivanidis (2019)[?] try to shed light to the relationship of economic activity and transportation infrastructure. However, their models only capture the direct effect of these improvements and the indirect effects due to reallocation or creation of economic activity. However, they do not capture the indirect effects coming from the change in commuting patterns across the entire transportation network that I show are significant.

The rest of the chapter is organized as follows: in section 2 I present the model of a transportation market in a city that features traffic congestion externalities. Next, in section 3 I expose the computational strategy to solve the model. Finally, in section 4 I illustrate the main properties of the model and compare it to a model without congestion externalities. Data and counterfactual exercises are left for the second chapter (2).

1.2 Example

Before developing the structural model I illustrate the main mechanism to generate traffic congestion and highlight the differences with a model where travel times are fixed by means of an example. In city A (see figure (1.2)) there are 200 commuters that need to complete the trip from origin O to destination D minimizing travel time. The task is to assign those commuters to the city's network under the assumption that they seek to minimize travel time.

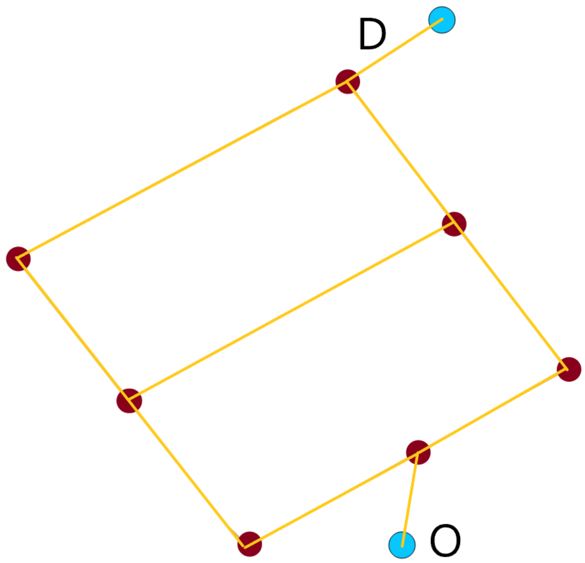
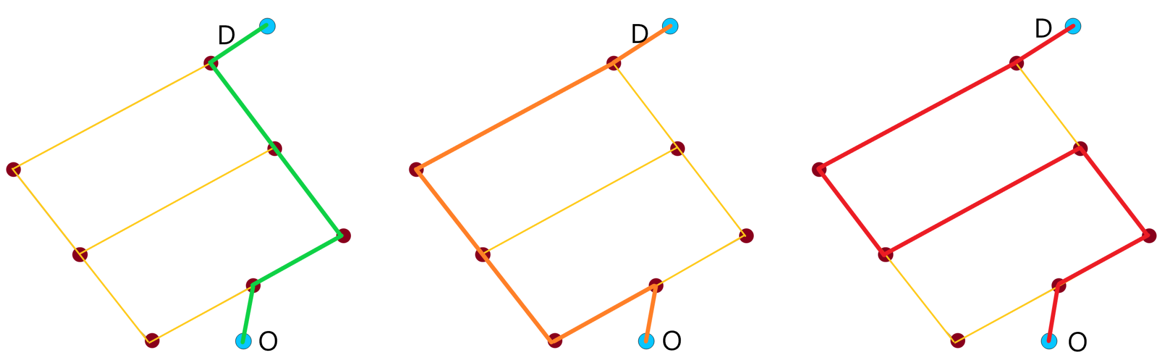


Figure 1.2: City A Network

In city A's network there are three different routes connecting O to D (see figure (1.3)). Each route i is characterized by its **free flow travel time**, t_0^i. This travel time is the minimum amount of time needed to complete the trip from O to D when no other car is present on the road. Moreover, each route i has attached a **congestion function** relating the mass of commuters using that route, m_i, and the route's travel time, t_i. For ease of exposition assume for now that this function is strictly increasing.

Figure 1.3: Routes in City A



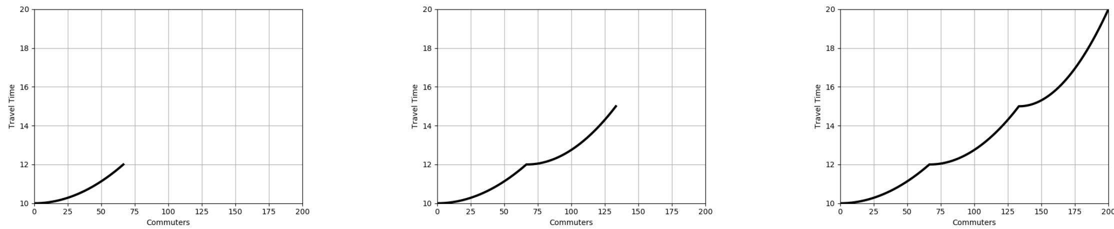
(a) Route 1 (b) Route 2 (c) Route 3

Route 1 (green, (1.3a)) is the shortest route and its free flow travel time is 10 minutes. The second route (orange, (1.3b)) has a free flow travel time of 12 minutes and the third route (red, (1.3c)) is the longest and the minimum amount of time needed to complete the route is 15 minutes.

In this example, commuters seek to minimize travel time and, since route 1 is the shortest, at first all chose route 1 (green) since travel time in this route is 10 minutes (subfigure 1.4a). However, if enough commuters use route 1 travel times will start to rise, because of the congestion function, and at some point will reach 12 minutes (subfigure 1.4b). At this point, route 2 (orange) is equally effective and hence, some commuters wills chose to use it instead of route 1. Finally, as commuters keep using these routes travel times will reach 15 minutes and route 3 (red) will start to be used (subfigure 1.4c). In equilibrium, all three routes are used and the travel time is equal to 20 minutes in each route. In general terms, **in equilibrium, if a route is used to connect O to D, it's travel time is equal to all other routes used to connect O to D, and equal to the minimum travel time between these two points**. See figure (1.4) to see the evolution of commuters and travel times.

In the case with no congestion externalities, travel times are fixed and don't depend on how many commuters are in the network. Hence, there is no feedback between network usage and travel times. In this example, if we fix travel times on a route to be equal to the route's free flow travel time, all commuters will chose to use route 1 since $t_0^1 < t_0^2 < t_0^3$.

Figure 1.4: Travel Times in City A as Function of Commuters



(a) Only Route 1 Used (b) Routes 1 & 2 Used (c) All Routes Used

Suppose now, that city A introduces a bus line connecting O to D (1.5). This line employs 14 minutes to connect O to D, travel time does not depend on the line's usage (think of a dedicated road lane) and there are no capacity constraints (everyone who wants to use the line can do it). Moreover, assume that we observe that the travel time to connect O to D by car is 20 minutes. That is, we observe the previous equilibrium where travel times arise as a function of network's use.

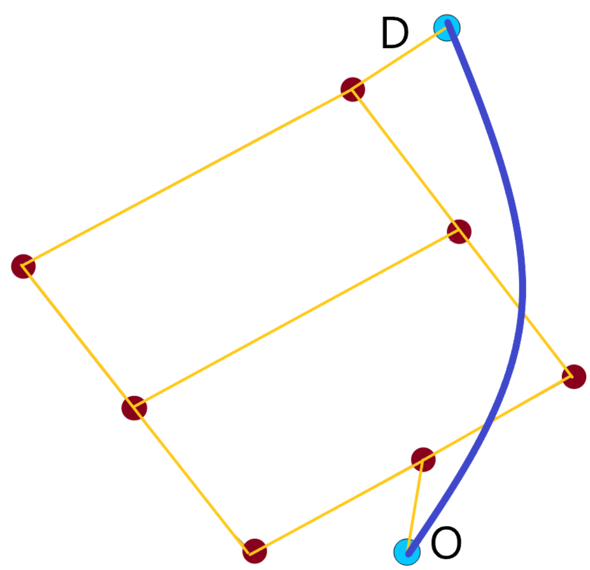


Figure 1.5: City A Network with Bus Line

Now, commuters compare this bus line to the 20 minutes they employ to commute by car. Now, some users will prefer to use the bus and car routes will start to lose commuters. Since commuters can use the bus and employ 14 minutes, any car route used has to be at most 14 minutes long. Hence, no one will use route 3 (red) since the minimum amount of time to cross it is 15 minutes. Car routes 1 and 2 together with the new bus line will be used. In equilibrium, commuters will distribute so that travel time from O to D is equal to 14 minutes.

In the absence of congestion externalities, when travel times are fixed, commuters will compare the 20 minutes employed to commute by car to the 14 minutes needed by the bus and choose the bus line. Since there is no feedback between road network use and travel times all commuters will choose the bus. Hence, when evaluating the impact of this new infrastructure the conclusions from a model with congestion externalities and a model without them will be very different.

1.3 Model

In this section I present a structural model of the transportation market in a city. On the demand side, commuters choose how to commute and the route from their residence to their working location taking as given mode-trip characteristics: travel time and monetary cost. Choices available are car, public transit, and walking. On the supply side, the road network is congestible, meaning that travel times depend on the number of users on the network. The more car users, the higher the travel times. However, travel times on the transit and street network do not depend on usage. Therefore, even if driving is the preferred choice of commuters, the more people using the road network increases car travel costs and makes other modes more attractive for commuters. Finally, an equilibrium in this model requires that given mode-trip characteristics commuters make choices that give raise to those mode-trip characteristics and that markets clear, that is, that every commuter gets to their destination.

1.3.1 Environment

I define a city as a set of discrete locations S. These locations are connected through a transportation network $\mathcal{G}$. To link locations o, $d \in S$, commuters have available three commuting modes $C = \{walk, bus, car\}$. Let $j \in C$ be a commuting choice. There are different routes connecting o to d for each mode choice. The set R^j_{od} collects all routes from o to d using mode j. A particular route is $r \in R^j_{od}$ and is just a collection of links L^r_{od}. Each link in the network is defined by the monetary cost and the time cost necesary to cross it, p_l and t_l respectively. Mode-route total travel time and money cost are:

$$t^{jr}_{od} = \sum_{l \in L^r_{od}} t_l \quad \text{and} \quad p^{jr}_{od} = \sum_{l \in L^r_{od}} p_l$$

There is an exogenous distribution of trips over locations in S. Let m_{od} be the mass of commuters in the city going from o to d. The matrix M is the commuting travel demand matrix that captures the mass of commuters assigned to each origin-destination pair:

$$M = \begin{pmatrix} m_{11} & \dots & \dots \\ \vdots & m_{od} & \dots \end{pmatrix}$$

1.3.2 Commuter's Problem

A commuter i is defined by a set of individual demographic characteristics, X_i, and an idiosyncratic commuting mode taste shock, ϵ_{ij}[4]. A commuter faced with trip od has to chose a commuting mode and a route to complete her trip. Her choice set is composed of all mode-route possible combinations connecting o to d, I denote that set $\Omega_{od} = \cup_{j \in C} R^j_{od}$.

Before making her choice $jr \in \Omega_{od}$, the commuter observes all mode-route characteristics: travel time, t^{jr}_{od}, and monetary cost, p^{jr}_{od}. Let $X^{jr}_{od} = \{t^{jr}_{od}, p^{jr}_{od}\}$ be the vector of such mode-route characteristics [5].

[4]Individual demographic characteristics include: age, gender, income, education...

[5]The set of trip characteristics can be extended to include walking time, number of transfers, etc.

The utility derived by commuter i faced with trip od from choice jr is then:

$$u_{iod}^{jr} = X_i\alpha^j + \beta^j + X_{od}^{jr}\beta + \epsilon_{ij} \tag{1.1}$$

where $\theta = \{\alpha^j, \beta^j, \beta\}$ is a set of parameters to be estimated. I allow demographic taste parameters to vary by commuting mode, α^j. This captures the fact that younger commuters may have a taste for public transit or that commuters with higher income may prefer to commute by car. Next, the parameter β^j is a mode specific taste parameter that captures the fact that, on average, a particular mode is preferred to other modes. Parameters associated to trip characteristics, β, translate these characteristics into utils. Finally, note that the idiosyncratic mode taste shock ϵ_{ij} only affects individual's preference for a given mode and does not affect the route choice. That is, commuters have preferences for a commuting mode but given the mode, they just care about minimizing travel costs and not about the route they take.

Let $X_{od}^{jr}\beta = \beta_t t_{od}^{jr} + \beta_p p_{od}^{jr}$ be the travel cost of mode-route jr.

With all that, the problem of a commuter is to choose a mode-route pair, jr, to maximize her utility taking as given mode-route characteristics. Formally,

$$\max_{jr\in\Omega_{od}} u_{iod}^{jr} = X_i\alpha^j + \beta^j + X_{od}^{jr}\beta + \epsilon_{ij} \tag{1.2}$$

Making the standard assumption of Type I Extreme Value errors, individual choice probabilities are given by:

$$p_{iod}(jr|X_i) = \frac{exp\{X_i\alpha^j + \beta^j + X_{od}^{jr}\beta\}}{\sum_{j\in C} exp\{X_i\alpha^j + \beta^j + X_{od}^{jr}\beta\}}$$

Now, integrating out demographic characteristics, I obtain mode-route shares for trip

I abstract from them for ease of exposition.

od:

$$s_{od}^{jr} = \int p_{iod}(jr|X_i)d\mathrm{F}(X_i)$$

Finally, the demand for a mode-route trip (od) is:

$$m_{od}^{jr} = s_{od}^{jr} m_{od} \tag{1.3}$$

Once the mass of commuters using each route is known I recover the mass of commuters using each link in the network. To do so, I define the following indicator function:

$$\delta_{odl}^{jr} = \begin{cases} 1 & \text{if } jr \in \Omega_{od} \text{ uses link } l \\ 0 & \text{otherwise.} \end{cases}$$

This function takes value 1 if mode-route jr uses link l to connect o to d. Summing over all origins, all destinations, and all possible routes I get the mass of commuters using a particular link:

$$m_l = \sum_{o \in S} \sum_{d \in S} \sum_{jr \in \Omega_{od}} \delta_{odl}^{jr} m_{od}^{jr} \tag{1.4}$$

Therefore, there is a mapping from travel costs to the mass of commuters using a link. That is: $m_l(t, p)$. Note that the use of link l depend not only on travel time and monetary cost of that particular link but all links on the network. Hence, if travel times and/or monetary cost of any link in the network changes, the mass of commuters using a particular link can potentially change.

1.3.3 Commuting Supply

On the supply side of the commuting market there is the city's transportation network. The network takes as input commuting demand and gives back travel costs. Here it is important to distinguish between the road network, that gets congested, and the public transit and street networks that do not. This assumption is further discussed in chapter 2 (2).

Let $\mathcal{G} = \{\mathcal{N}, \mathcal{L}\}$ be the city's transportation network. Each $n \in \mathcal{N}$ represents a location in the city or the intersection of links connecting locations. Hence the set of all origins and destinations, S, is a subset of all the network's nodes, $S \subset \mathcal{N}$. Links $l \in \mathcal{L}$ represent road, street or rail segments. Each link in the network is defined by it's capacity, c_l, and free-flow travel time, t_l^0. Free-flow travel time is the minimum amount of time required to traverse the link with no other commuter on the link. The difference between the public (street plus public transit) and the road network is that the latter is congestionable. That is, time and cost depend on how many commuters are on the network and their distribution. To introduce this characteristic let me distinguish between the road network, $\mathcal{G}^R$, and the public network, $\mathcal{G}^P$.

Links in the road network have attached a congestion function relating link usage, m_l, to travel time and cost:

$$t_l = t_j(m_l|c_l, t_l^0)$$

$$p_l = p_j(m_l|c_l, t_l^0)$$

This functions are nonnegative, single-valued, monotonically increasing and strictly convex. For a more thorough discussion of this functions and how to estimate their parameters see chapter 2 below (2).

Equation (1.4) gives the flow of commuters for every link and route costs can be recovered as the sum of link costs. Hence, travel time and cost of route r connecting o to d using route r are:

$$t_{od}^{jr} = \sum_{l \in L_{od}^r} t_j(m_l|c_l, t_l^0) \quad \text{and} \quad p_{od}^{jr} = \sum_{l \in L_{od}^r} p_j(m_l|c_l, t_l^0) \tag{1.5}$$

On the other hand, routes on the public network $\mathcal{G}^P$ do not get congested and hence, trip characteristics are independent of network usage. Hence, for non-car travel modes travel time is:

$$t_{od}^{jr} = \sum_{l \in L_{od}^r} t_l^0$$

Regarding money cost, I assume that commuters that chose to walk do not incur any cost and that public transit costs are given by the city's fare scheme.

Finally, note that, while the public network is not affected by the flow of commuters, the road network is. Equation (1.5) gives a mapping from road network use to trip characteristics.

1.3.4 Equilibrium

The city's commuting market is in equilibrium when, for a given set of commuting flows the travel times and monetary costs are such that commuters choices give rise to the same set of traffic flows. Moreover, the equilibrium requires that markets clear, that is, the travel demand for each origin-destination pair is equal to the sum of travel flows across transportation modes and routes.

Commuting Market Equilibrium Given travel demand, M, an equilibrium is a set of mode-route commuters, $\{m^{jr*}\}_{jr\in\Omega_{od}}\ \forall o, d \in S$, travel times $\{t^{jr*}\}_{jr\in\Omega_{od}}\forall o, d \in S$ and travel costs $\{p^{jr*}\}_{jr\in\Omega_{od}}\forall o, d \in S$, such that:

- Given travel times and costs $\{t^{jr*}, p^{jr*}\}_{jr\in\Omega_{od}}\forall o, d \in S$ equilibrium mode-route commuters are $\{m^{jr*}(t^*, p^*)\}_{jr\in\Omega_{od}}\ \forall o, d \in S$
- Given equilibrium mode-route mass of commuters $\{m^{jr*}\}_{jr\in\Omega_{od}}\ \forall o, d \in S$, travel times and cost are $\{t^{jr*}(m^*)\}_{jr\in\Omega_{od}}\forall o, d \in S$ and $\{p^{jr*}(m^*)\}_{jr\in\Omega_{od}}\forall o, d \in S$ respectively.
- Market clearing: $m_{od} = \sum_{jr\in\Omega_{od}} m_{od}^{jr} \quad \forall o, d \in S$.

Equilibrium Computation

The equilibrium computation of the model is based in finding a fixed point between mode-route choices and trip characteristics. In each iteration I have to solve the commuter's mode choice problem and the car routing problem. The assumption that walking and public transit travel time and money costs are independent of transportation network use allow me to abstract from the walking and public transit routing problem in every iteration. Commuters simply choose the route that minimizes travel costs, which is the same in each iteration.

The strategy to solve for the model's equilibrium is the following: first, given a set of travel costs, commuters make their mode choice. Second, given commuting demand for the different modes, the routing problem is solved: car routing is solved using the *user equilibrium (UE)* (see algorightm (**??**)) and walking and transit problems simply reduce to a minimum cost path solved in the first iteration. Third, I compare travel costs with the new usage to the previous travel costs. If they are the same an equilibrium is found. If not, go back to the first step.

In the reminder of this section I explain in more detail the algorithms used to solve the model and the routing problem in the road network.

Model Equilibrium Algorithm

This algorithm takes as inputs: origin-destination demand matrix, walking and bus travel times and monetary costs, and the road network. Gives as output: equilibrium car travel times and money costs, and equilibrium commuting mode-route flows that satisfy the conditions of definition (1.3.4).

Algorithm 1 Commuting Market Equilibrium

Step 0: *Initialization* Perform all-or-nothing assignment based on $t_l = t_l(0)$, $\forall l$. Obtain free-flow travel times for each origin-destination pair. Set t^{carn}. Set counter $n = 1$

Step 1: *Split Demand* Divide demand between commuting modes based on T^{bus}, T^{walk}, and T^{carn}. Obtain $M^{car} = s^{car}M$.

Step 2: *Assign Car Demand* Assign car demand M^{car} to the road network using UE algorithm (**??**). Obtain car flows m_l, $\forall l$.

Step 3: *Update Travel Times* Obtain new travel times as:

$$t_l^{carn+1} = t(m_l) \quad \forall l$$

And recover origin-destination travel times T^{n+1}

Step 4: *Convergence Test* If a convergence criterion is met, stop and set $\{T^{n+1}\}$ as the solution. Otherwise set $n = n + 1$ and go back to *Step 1.* =0

The algorithm to solve for the model's equilibrium starts (step 0) with by finding the free-flow travel times for all origin-destination pairs. Free-flow travel time is the travel time a motorist would travel if there were no congestion on the road. This is accomplished by performing all-or-nothing assignment (see subsection (1.3.4)).

The next step (step 1) is to split travel demand into the different commuting modes based on *bus*, *walk* and *car* travel times and monetary costs using equation (1.3). Notice that *bus* and *walk* travel times are fixed and known and are not actualized. Once the demand is split the, next step (step 2) is to assign car demand, M^{carn}, to the road network. To do so I use the User Equilibrium Traffic Assignment algorithm (see algorithm (**??**)). The output of this algorithm are flows and travel times and monetary costs per link. Hence, we can recover origin-destination travel times and monetary costs (step 4) using equation (1.5) and check if some convergence criterion is met (step 5). If this is the case, those travel times and monetary costs are equilibrium travel times and money costs and commuting mode shares are also fixed. If the convergence is not met then we go back to split demand (step 1) and perform another iteration.

Car Routing Problem: User Equilibrium algorithm

In this subsection I sketch how to compute the equilibrium in the car routing problem given car travel demand, M^{car}, and the road network of the city, $\mathcal{G}^R = \{\mathcal{N}^R, \mathcal{L}^R\}$. This is mainly a technical section that considers the definition and computation of the traffic assignment problem in the road network. The reader not interested in the details can skip it. For ease of exposition I abstract of the transportation mode subscript, j. I present the problem when only travel time is considered since monetary travel costs are just a function of length and cents per unit of length.

For each pair of locations o and d there is a demand for travel m_{od} which can be compactly expressed in a matrix M^{car}. Given M^{car}, the problem is to assign these travel demands[6] to the road network $\mathcal{G}^R = \{\mathcal{N}^R, \mathcal{L}^R\}$. The solution to this problem is based on Wardrop's first principle[7]: *the travel time on all routes used from o to d is equal and less than the time in any other route used by a single vehicle.* That is, no user can be better off by deviating from the optimal strategy given the routes taken by all other users.

For any pair of locations $o, d \in S \subseteq \mathcal{N}^R$ let R_{od} be the set of all possible (non-cyclical) car routes from residence location o to employment location d. Any route $r \in R_{od}$ has associated some positive flow, h^r_{od}, if the route is used and zero otherwise. Let $\mathbf{h} = \{h^r_{od}\}_{\forall o,d,r}$ be the flow distribution over all possible origin-destination pairs and all routes. Then, for any link $l \in \mathcal{L}^R$ the flow in that link because of travel from o to d, m_{lod}, is: cuidado con esta equacion

$$m_{lod} = \sum_{r \in R_{od}} \delta^r_{odl} h^r_{od} \tag{1.6}$$

[6]For background on traffic assignment problems see Sheffi (1984) ([**?**]) or Florian and Hearn (2001) ([**?**])

[7]More on the relation of Wardrop and Nash equilibrium concepts see ([**?**])

where

$$\delta^r_{odl} = \begin{cases} 1 & \text{if } r \in R_{od} \text{ uses link } l \\ 0 & \text{otherwise} \end{cases} \tag{1.7}$$

and the total flow in link l, m_l, is:

$$m_l = \sum_{o \in S} \sum_{d \in S} m_{lod} \tag{1.8}$$

Notice that this is a 'car only' version of equation (1.4). Now, for any route $r \in R_{od}$ let τ^r_{od} be the time cost associated with transitioning the route. From the above definitions, this cost is a function of the whole distribution of travel flows on the entire network, $\mathbf{h}$. Then:

$$\tau^r_{od}(\mathbf{h}) = \sum_{l \in \mathcal{L}^R} \delta^r_{odl} t_l(m_l | t^0_l, c_l) \tag{1.9}$$

Finally, for any $r \in R_{od}$ let λ_{od} be the shortest path, in terms of time, from residence location o to employment location d. That is,

$$\lambda_{od} = \min_{r \in R_{od}} \tau^r_{od} \tag{1.10}$$

With all this notation, the equilibrium of the road traffic assignment is formally:

Definition User Equilibrium: Given travel demand matrix D, a user equilibrium in the road network $\mathcal{G}^R = \{\mathcal{N}^R, \mathcal{L}^R\}$ is characterized by the following conditions:

$$h_{odr}(\tau_{odr} - \lambda_{od}) = 0, \quad \forall r \in \mathcal{R}_{od}, \forall o, d \in S \tag{1.11}$$

$$\tau_{odr} - \lambda_{od} \geq 0, \quad \forall r \in \mathcal{R}_{od}, \forall o, d \in S \tag{1.12}$$

$$\sum_{r \in \mathcal{R}_{od}} h_{odr} = m_{od}, \quad \forall o, d \in S \tag{1.13}$$

$$h_{odr} \geq 0, \quad \forall r \in \mathcal{R}_{od}, \forall o, d \in S \tag{1.14}$$

$$\lambda_{od} \geq 0, \quad \forall o, d \in S \tag{1.15}$$

Equations (1.11) and (1.12) state that if in a route there is positive flow it is because its travel time is equal to the shortest path travel time while it has zero flow if the travel time is bigger than the shortest path travel time. Equation (1.13) states that the flow on all routes from o to d adds up to the total demand of travel from o to d, market clearing. Finally, equations (1.14) and (1.15) ensure nonnegativity of route flows and travel times.

Letting $\mathbf{m} = \{m_l\}_{\forall l \in \mathcal{L}^R}$ be the vector of link flows from equation (1.8), Beckmann ([**?**]) showed that the first order conditions of the following optimization problem are equivalent to the conditions for a user equilibrium to the traffic assignment problem:

$$\begin{aligned} \min_{\mathbf{m}} \qquad & T(\mathbf{m}) = \sum_{l \in \mathcal{L}^R} \int_0^{m_l} t_l(\omega | t_l^0, c_l) d\omega \\ \text{subject to} \qquad & \sum_{r \in R_{od}} h_{od}^r = m_{od}, \quad \forall o, d \in S \\ & h_{od}^r \geq 0, \quad \forall r \in R_{od}, \forall o, d \in S \\ & \sum_{o \in S} \sum_{d \in S} \sum_{r \in R_{od}} \delta_{odl}^r h_{od}^r = m_l, \quad \forall l \in \mathcal{L}^R \end{aligned} \tag{1.16}$$

Let the equilibrium distribution of flows over links be $\mathbf{m}^*$, then the equilibrium travel times in each link is $t_l^* = t_l(m_l^* | t_l^0, c_l)$. Equilibrium travel time from origin o to destination d is then:

$$\tau_{od}^* = \sum_{l \in \mathcal{L}^R} \delta_{odl}^r t_l(m_l^* | t_l^0, c_l) \tag{1.17}$$

Note that for each origin-destination pair there is only one travel time since, in equilibrium, all routes with positive flow have the same travel time. Let τ^* be the matrix of equilibrium travel times.

The following algorithm is an application of the Frank-Wolfe convex-combination algorithm to solve problem (1.16). Inputs are: car travel demand (origin-destination matrix), M^{car}, and a road network, $\mathcal{G} = \{\mathcal{N}, \mathcal{L}\}$. Outputs are: link flows, $\mathbf{m}^*$, and then car travel times, τ^*, can be recovered using equation (1.17).

Algorithm 2 User Equilibrium of the Traffic Assignment

Require:

Ensure:

Step 0: *Initialization* Perform all-or-nothing assignment based on $t_l = t_l(0)$, $\forall l$. This yields $\{m_l^1\}$. Set counter $n = 1$.

Step 1: *Update* Set $t_l^n = t_l(m_l^n)$, $\forall l$.

Step 2: *Direction Finding* Perform all-or-nothing assignment based on $\{t_l^n\}$. This yields a set of (auxiliary) flows $\{y_l^n\}$.

Step 3: *Line Search* Find α_n that solves

$$\min_{0 \geq \alpha \geq 1} \int^{m_l^n + \alpha(y_l^n - m_l^n)} t_l(\omega) d\omega$$

Step 4: *Move* Set

$$m_l^{n+1} = m_l^n + \alpha_n (y_l^n - m_l^n) \quad \forall l$$

Step 5: *Convergence Test* If a convergence criterion is met, stop and set $\{m_l^{n+1}\}$ as the solution. Otherwise set $n = n + 1$ and go back to *Step 1*. =0

All-or-Nothing Traffic Assignment

In the all-or-nothing assignment travel times don't depend of flows and $t_l(m_l) = t_l'$ where t_l' is fixed and known. The problem here is to find the flow pattern that minimizes the total travel time over the network, given the (fixed and known) values of the link travel times and the M^{car} matrix. The solution of this problem is conceptually straightforward: all the flow for a given *od* pair is assigned to the minimum travel time path connecting this pair. All other paths connecting this *od* pair do not carry flow. Consequently, this traffic assignment procedure is known as the "all-or-nothing" assignment. The resulting flow pattern is both an equilibrium situation (since no user will be better off by switching paths) and an optimal assignment (since the total travel time in the system is obviously minimized)[8].

[8] I use Dijkstra's shortest path algorithm to compute the all-or-nothing assignment

1.4 Model Simulation

To illustrate the properties of the model and compare it to a model that does not account for congestion externalities I run the model in the very simplified Sioux Falls, South Dakota, network.

The Sioux Falls road network is composed of 24 nodes and 76 links (see figure (1.6) below. Red links do not belong to the original Sioux Falls network.). Each of those 24 nodes is at the same time an origin and a destination node, a 24 origin-destination travel demand matrix is given (see table (2.12) below). Links differ in their free flow travel time, t_0, and capacity, c_l. Those values are observed from the data. Each link has attached a congestion function relating link's commuting flow to travel time. In particular, the function used is the one proposed by the Bureau of Public Roads (BPR) in 1964:

$$t_l = t_0\Big(1 + \alpha \frac{m_l}{c_l}^{\beta}\Big)$$

with parameters $\alpha = 0.15$ and $\beta = 4$. This function satisfies the requirements stated in section (1.3): is non-negative (the minimum value is equal to the free flow travel time), increasing and strictly convex.

The original Sioux Falls problem is focused on traffic assignment and does not involve a mode choice problem. The network represents the road network. To overcome this problem I use the following strategy: first, I solve the traffic assignment problem using all-or-nothing assignment using free flow travel times, next I increase each origin-destination travel time by a random factor ranging from 0 to 30%. I call this origin-destination matrix of travel times public transit travel times.

In this example commuters have only two commuting modes available: $j \in \{\text{bus, car}\}$.

Commuter i with trip od derives utility from mode-route jr from the following utility function:

$$u_{iod}^{jr} = \beta^j + \beta_{tt} tt_{od}^{jr} + \epsilon_i$$

where tt_{od}^{jr} is the travel time in mode-route jr to complete trip od. Parameter β_{tt} converts travel minutes into utils and β^j is a mode specific taste parameter. This utility function is a simplification of equation (utility) where the only trip characteristic is travel time and the only individual characteristic is the idiosyncratic taste shock ϵ_i. Table (1.1) shows parameter values used in the simulation.

Table 1.1: Simulation Utility Function Parameters

Parameter	Description	Value
β_{tt}	Travel Time	-0.27
β^{car}	Car Taste	1.3
β^{bus}	Bus Taste	0.7

The strategy I use in this simulation is the following: first, I compute the equilibrium of the mode-route choice problem in the original network. I call this equilibrium the *benchmark equilibrium*. Next, I make an improvement on the network and reevaluate the equilibrium in the case with congestion externalities (*congestion equilibrium*), and in the case without congestion (*myopic equilibrium*). Finally, I compare the results to highlight the main points of the model with congestion and how they diverge from those where commuters are myopic.

In the benchmark equilibrium (column 1 in table (1.2) mode shares are 62.5% and 37.5% for car and bus respectively. This is as expected since, everything else equal, commuters prefer to use the car than the bus ($\beta^{car} > \beta^{bus}$). Total system travel time is of 60,957 hours, this is the sum of travel time employed by each commuter to complete their trip. Finally, total welfare is a negative number since travel time generates desutility. Because of the choice of parameters, if car travel time is higher than 4.8 minutes, utility of car users is negative. This number reduces to 2.6 minutes in the case of bus users.

Next I introduce an improvement in the public transit network. The way in which I model it consists on making the connection between two nodes faster. Then, I recalculate all public transit travel times under this new network while keeping the road network unchanged.

The connection between nodes 11 and 13 is improved by adding two new links to the public transit network. Links 77 and 78 are highlighted in red in figure (1.6). I recalculate bus travel times in this new network. Out of 576 posible origin-destination pairs in the city, public transit travel time decreases in only 16 trips. This means that the network improvement affects 2.7% of all public transit bilateral commutes.

Now I recalculate the equilibrium under this new scenario and in the presence of congestion externalities. Results of the *congestion equilibrium* are reported in column 2 of table (1.2). Car share drops to 2 percentage points (60.5%) and bus share increases by 2 percentage points (39.5%) relative to the benchmark equilibrium. The introduction of two new links in the public transit network affects 16 bilateral public transit commutes and those, because of the congestion externalities, translate into 373 bilateral car trips affected (around 65% of total bilateral commutes). Since commuters observe that travel times are lower in the bus network some change their choice. This empties the road network and commuters reroute accordingly. This feedback loop between mode choice and travel times continues until there is no profitable deviation by any commuter. Hence, in the presence of congestion externalities, the effect of the public transit improvement expands to all the transportation network.

As a result of this shift in mode shares and redistribution of commuters on the network, total system travel time increases by 404 hours and welfare decreases by 0.16%. This is another property of the model, commuters when making their choices do not internalize the travel cost they impose in all other commuters and an improvement in the network does not need to be welfare improving.

In the case where travel times are exogenous and don't depend on the network use, *myopic equilibrium*, car share decreases by 0.75 percentage points and bus share increases by the same amount. In this case, since the choice of the commuters faced with one of those 16 bilateral commutes affected by the network improvement don't affect the choices of other commuters the feedback loop is not initialized. Therefore, the effect doesn't expand through the network and only 16 commutes are affected. Moreover, in this case, any improvement will reduce travel time (433 hours less in the myopic equilibrium) and increase welfare (+0.9%).

Table 1.2: Simulation Equilibrium Results

	Benchmark	Congestion	
		Yes	No
Car Share	62.53	60.49	61.78
Bus Share	37.47	39.51	38.22
Commutes Affected	.	373	16
Total Time (hours)	60,957.46	61,361.65	60,524.03
Welfare	-599,804.18	-600,762.46	-594,397.48

The simplified example of Sioux Falls illustrates the main properties of the model: first, in the presence of congestion externalities, the effect of changes in the transportation network extend (potentially) to the whole network. Next, an intervention in the network does not need to be welfare improving because commuters do not internalize the cost they impose in other commuters with their choices. Finally, evaluating such interventions in the presence of congestion externalities can lead to dramatically different conclusions to the case without externalities.

Appendix

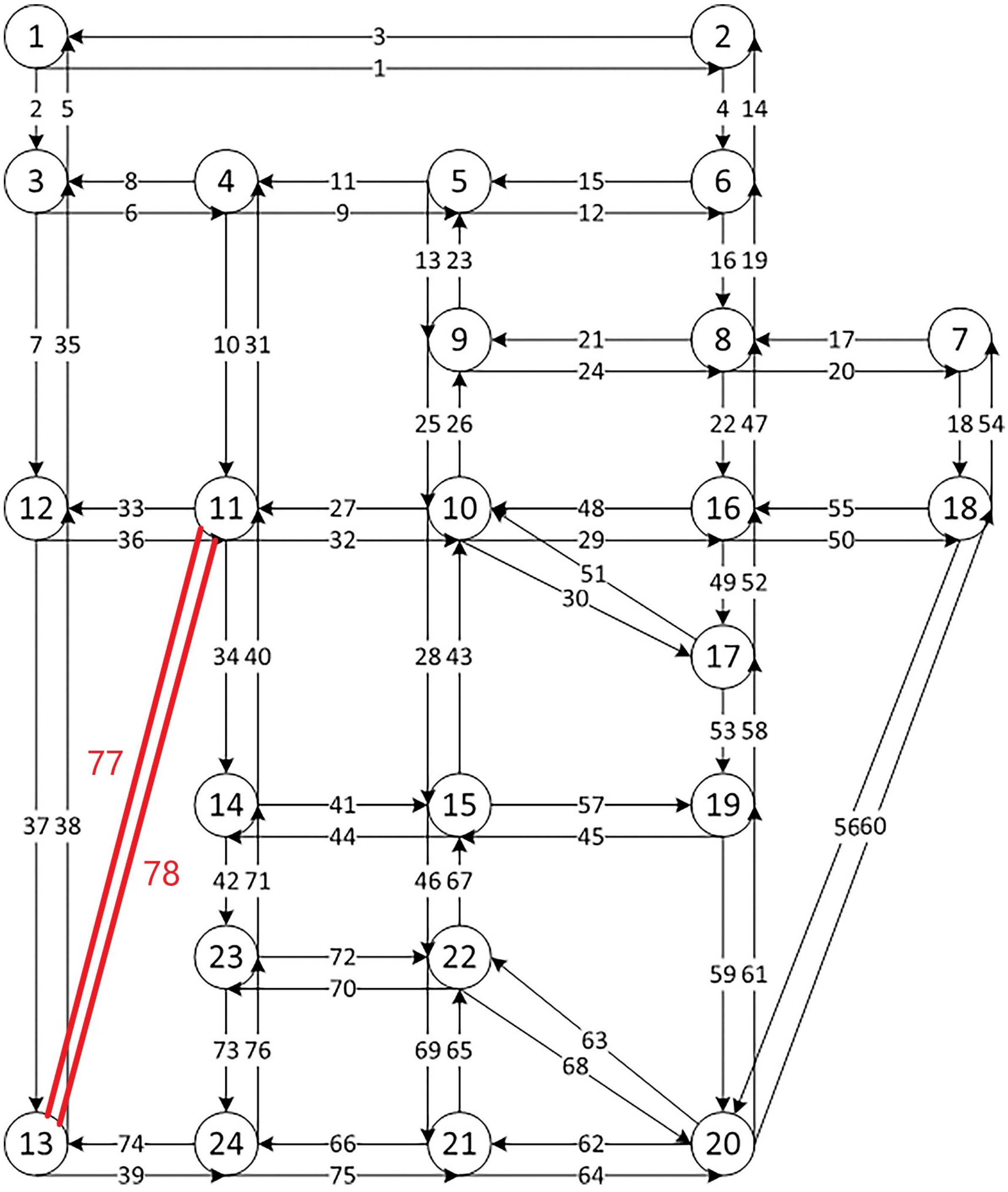


Figure 1.6: Sioux Falls, SD, road network

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
1	0.0	100.0	100.0	500.0	200.0	300.0	500.0	800.0	500.0	1300.0	500.0	200.0	500.0	300.0	500.0	500.0	400.0	100.0	300.0	300.0	100.0	400.0	300.0	100.0
2	100.0	0.0	100.0	200.0	100.0	400.0	200.0	400.0	200.0	600.0	200.0	100.0	300.0	100.0	100.0	400.0	200.0	0.0	100.0	100.0	0.0	100.0	0.0	0.0
3	100.0	100.0	0.0	200.0	100.0	300.0	100.0	200.0	100.0	300.0	300.0	200.0	100.0	100.0	100.0	200.0	100.0	0.0	0.0	0.0	0.0	100.0	100.0	0.0
4	500.0	200.0	200.0	0.0	500.0	400.0	400.0	700.0	700.0	1200.0	1400.0	600.0	600.0	500.0	500.0	800.0	500.0	100.0	200.0	300.0	200.0	400.0	500.0	200.0
5	200.0	100.0	100.0	500.0	0.0	200.0	200.0	500.0	800.0	1000.0	500.0	200.0	200.0	100.0	200.0	500.0	200.0	0.0	100.0	100.0	100.0	200.0	100.0	0.0
6	300.0	400.0	300.0	400.0	200.0	0.0	400.0	800.0	400.0	800.0	400.0	200.0	200.0	100.0	200.0	900.0	500.0	100.0	200.0	300.0	100.0	200.0	100.0	100.0
7	500.0	200.0	100.0	400.0	200.0	400.0	0.0	1000.0	600.0	1900.0	500.0	700.0	400.0	200.0	500.0	1400.0	1000.0	200.0	400.0	500.0	200.0	500.0	200.0	100.0
8	800.0	400.0	200.0	700.0	500.0	800.0	1000.0	0.0	800.0	1600.0	800.0	600.0	600.0	400.0	600.0	2200.0	1400.0	300.0	700.0	900.0	400.0	500.0	300.0	200.0
9	500.0	200.0	100.0	700.0	800.0	400.0	600.0	800.0	0.0	2800.0	1400.0	600.0	600.0	600.0	900.0	1400.0	900.0	200.0	400.0	600.0	300.0	700.0	500.0	200.0
10	1300.0	600.0	300.0	1200.0	1000.0	800.0	1900.0	1600.0	2800.0	0.0	4000.0	2000.0	1900.0	2100.0	4000.0	4400.0	3900.0	700.0	1800.0	2500.0	1200.0	2600.0	1800.0	800.0
11	500.0	200.0	300.0	1500.0	500.0	400.0	500.0	800.0	1400.0	3900.0	0.0	1400.0	1000.0	1600.0	1400.0	1400.0	1000.0	100.0	400.0	600.0	400.0	1100.0	1300.0	600.0
12	200.0	100.0	200.0	600.0	200.0	200.0	700.0	600.0	600.0	2000.0	1400.0	0.0	1300.0	700.0	700.0	700.0	600.0	200.0	300.0	400.0	300.0	700.0	700.0	500.0
13	500.0	300.0	100.0	600.0	200.0	200.0	400.0	600.0	600.0	1900.0	1000.0	1300.0	0.0	600.0	700.0	600.0	500.0	100.0	300.0	600.0	600.0	1300.0	800.0	800.0
14	300.0	100.0	100.0	500.0	100.0	100.0	200.0	400.0	600.0	2100.0	1600.0	700.0	600.0	0.0	1300.0	700.0	700.0	100.0	300.0	500.0	400.0	1200.0	1100.0	400.0
15	500.0	100.0	100.0	500.0	200.0	200.0	500.0	600.0	1000.0	4000.0	1400.0	700.0	700.0	1300.0	0.0	1200.0	1500.0	200.0	800.0	1100.0	800.0	2600.0	1000.0	400.0
16	500.0	400.0	200.0	800.0	500.0	900.0	1400.0	2200.0	1400.0	4400.0	1400.0	700.0	600.0	700.0	1200.0	0.0	2800.0	500.0	1300.0	1600.0	600.0	1200.0	500.0	300.0
17	400.0	200.0	100.0	500.0	200.0	500.0	1000.0	1400.0	900.0	3900.0	1000.0	600.0	500.0	700.0	1500.0	2800.0	0.0	600.0	1700.0	1700.0	600.0	1700.0	600.0	300.0
18	100.0	0.0	0.0	100.0	0.0	100.0	200.0	300.0	200.0	700.0	200.0	200.0	100.0	100.0	200.0	500.0	600.0	0.0	300.0	400.0	100.0	300.0	100.0	0.0
19	300.0	100.0	0.0	200.0	100.0	200.0	400.0	700.0	400.0	1800.0	400.0	300.0	300.0	300.0	800.0	1300.0	1700.0	300.0	0.0	1200.0	400.0	1200.0	300.0	100.0
20	300.0	100.0	0.0	300.0	100.0	300.0	500.0	900.0	600.0	2500.0	600.0	500.0	600.0	500.0	1100.0	1600.0	1700.0	400.0	1200.0	0.0	1200.0	2400.0	700.0	400.0
21	100.0	0.0	0.0	200.0	100.0	100.0	200.0	400.0	300.0	1200.0	400.0	300.0	600.0	400.0	800.0	600.0	600.0	100.0	400.0	1200.0	0.0	1800.0	700.0	500.0
22	400.0	100.0	100.0	400.0	200.0	200.0	500.0	500.0	700.0	2600.0	1100.0	700.0	1300.0	1200.0	2600.0	1200.0	1700.0	300.0	1200.0	2400.0	1800.0	0.0	2100.0	1100.0
23	300.0	0.0	100.0	500.0	100.0	100.0	200.0	300.0	500.0	1800.0	1300.0	700.0	800.0	1100.0	1000.0	500.0	600.0	100.0	300.0	700.0	700.0	2100.0	0.0	700.0
24	100.0	0.0	0.0	200.0	0.0	100.0	100.0	200.0	200.0	800.0	600.0	500.0	700.0	400.0	400.0	300.0	300.0	0.0	100.0	400.0	500.0	1100.0	700.0	0.0

Table 1.3: Origin-Destination Demand Matrix For Sioux Falls, SD

Chapter 2

Evaluating Public Infrastructure Investments in the Presence of Traffic Congestion Externalities: The Case of METRO LA

2.1 Introduction

In this chapter I bring the model presented in chapter 1 (1) to the data and use it to evaluate the impact of the Expo rail line extension in Los Angeles county.

To estimate the demand side parameters I use the 2012 California Household Travel Survey (CHTS-2012). The survey provides data on commuting mode choice and the main trip characteristics together with individual demographics. However, it does not provide trip characteristics for commuting modes not chosen by the commuter. To overcome this problem I extend the CHTS data with trip characteristics coming from requests to Google Maps' API. This allows me to have data on the mode choice and on trip characteristics of all commuting modes. Using demand parameter estimates I find that the Value of Time (VoT) is around 20$ per hour.

On the supply side, first of all I present the Bureau of Public Roads (BPR) congestion function. This function relates flow of users on a network link to the time spent crossing that link, and satisfies all the requirements stated in the model presented in the first chapter (1). To estimate this function's parameters I use Caltrans Performance Measurement System (PeMS) dataset. I use more than 3 million hourly flow of cars and average speed observations in different segments of the major freeways in Los Angeles county. The parameters I estimate for Los Angeles County imply that congestion kicks in for lower levels of vehicle flow than previously thought and implied by the parameter values suggested by the Bureau of Public Roads.

Next, I test model's performance to the observed data in the CHTS-2012. To do so I simulate a typical weekday 8am commute. Before running the model I need data on travel demand and the transportation network. The morning commute travel demand is obtained by a combination of Longitudinal Origin-Destination Employment Statistics (LODES) and American Community Survey (ACS) data. On the transportation network side, the road network comes from the Southern California Association of Governments (SCAG). To simulate the public transit and road network I use a mix of Open Trip Planner (OTP) and General Transportation Feed Specification (GTFS). Then I simulate the morning commute and compare it to the data. The model replicates accurately the main moments in the data: mode shares and average travel times.

Finally I perform two different counterfactuals. The first conterfactual evaluates the effect of the Expo rail line in Los Angeles county. I find that the county saves per year $235.3M when accounting for congestion versus $561.4M when congestion is not accounted for. In the second counterfactual I estimate the welfare gains of the whole rail system in Los Angeles County to be $1.9B per year.

The rest of the chapter is organized as follows: in section 2 and 3 I estimate the demand and supply side of the model respectively. Section 4 presents the data to simulate the model and section 5 assesses model performance. Section 6 estimates the value of the Expo rail line in Los Angeles county and section 7 estimates the value of the entire rail network. Finally, section 8 concludes.

2.2 Demand Estimation

In this section I show how to estimate demand side parameters of the model presented in the previous chapter (1). The utility of commuter i with trip od derived from mode-route alternative jr is:

$$u_{iod}^{jr} = X_i \alpha^j + \beta^j + X_{od}^{jr} \beta + \epsilon_{ij} \tag{2.1}$$

where X_i is a set of observable individual demographic characteristics, and X_{od}^{jr} is a set of observable mode-route characteristics. The term ϵ_{ij} is a mode specific idiosyncratic taste shock. The parameters to be estimated are $\theta = \{\alpha^j, \beta^j, \beta\}$.

2.2.1 Choice Data

The main source of consumer choice data for the demand estimation is the 2012 California Household Travel Survey (CHTS-2012) conducted by the California Department of Transportation (CalTrans) over the 2010-12 period. The survey provides detailed travel diaries of nearly 42.500 households in the state of California together with household and individual demographics[1]. Travel diaries provide the main source of observable trip characteristics for the realized choices: travel times, walking distance, and wait times.

A travel diary starts on the assigned day at 3am. Then, once the person moves from her starting location has to report start time, end time, mode, duration and purpose

[1]Individual demographic characteristics include age, gender, education, occupation, etcetera. Household characteristics include household members, income level, number of cars available among others

of the trip for each change of location. What makes difficult to identify trips is that a transfer counts as a trip, even if the final destination is a work location. Therefore, a survey trip is not equivalent as a model trip.

As an example, consider the travel diary of person number 2 in household 1044767 (figure (2.1)). The travel diary was recorded on a Tuesday and the residence and work census tract are 294701 and 11300 respectively. I observe 15 rows and 14 trips entries in this person's travel diary. The travel diary starts at 3am at the residential tract. At 5am this person walks for 3 minutes and 0.24 miles ($mode = 1$, $tripdur = 3$, and $tripdistance = 0.24$) with the purpose of transferring to a different transportation mode ($apurp = 21$). Waits for 12 minutes (from 5:03 am to 5:15am) when takes a local bus ($mode = 15$) and commutes for 20 minutes and 5.6 miles. After 5 more transfers this person walks 5 minutes and 0.59 miles to her work census tract with the purpose of working. For the CHTS this person made 7 trips, however, this is just one working trip according to the model. Hence, to identify model trips I have to sum trip duration and distance across all intermediate trips. 79 minutes and 38.78 miles.

After her workday this person starts their trip back home at 4:35pm and needs 7 more trips to go back home. 96 minutes and 43.06 miles.

	isampn	dow	hctract	perno	wtract	mode	tripdur	tripdistance	tract	tripno	apurp	stime	etime
1	1044767	2	294701	2	11300	.	.	.	294701	.	1	03:00	05:00
2	1044767	2	294701	2	11300	1	3	.2425624	40204	1	21	05:03	05:15
3	1044767	2	294701	2	11300	15	20	5.638538	964600	2	21	05:35	05:36
4	1044767	2	294701	2	11300	15	1	.6957301	962501	3	21	05:37	05:50
5	1044767	2	294701	2	11300	15	35	11.84629	30402	4	21	06:25	06:55
6	1044767	2	294701	2	11300	15	5	15.89419	961600	5	21	07:00	07:05
7	1044767	2	294701	2	11300	15	10	3.873589	400	6	21	07:15	07:50
8	1044767	2	294701	2	11300	1	5	.5939068	11300	7	9	07:55	16:30
9	1044767	2	294701	2	11300	1	5	.5939068	400	8	21	16:35	16:45
10	1044767	2	294701	2	11300	15	15	3.860528	961600	9	21	17:00	17:05
11	1044767	2	294701	2	11300	15	30	20.10298	30402	10	21	17:35	17:55
12	1044767	2	294701	2	11300	15	20	12.12873	962501	11	21	18:15	18:16
13	1044767	2	294701	2	11300	1	2	.5500982	964600	12	21	18:18	18:25
14	1044767	2	294701	2	11300	15	20	5.583721	40204	13	21	18:45	18:46
15	1044767	2	294701	2	11300	1	4	.2420616	294701	14	1	18:50	21:30

Figure 2.1: Travel Diary

In the final sample I only consider residence-work trips (or vice versa) since commuters have less margin to modify their trips in terms of start time and direction. In a shopping trip commuters can change the destination and/or trip time or even the weekday. Hence, they have more margins to adjust than just the commuting mode.

I define public transit trips as any trip that uses the public transit system and walking trips only involve walking. I discard all observations that use car and public transit in the same trip (park & ride type of combinations) since this is not allowed in the model and represent less than 0.3% of all trips.

After identifying all trips, the final sample is composed of 28.000 work weekday trips. Table (2.1) presents descriptive statistics of the CHTS final sample I use in the estimation. Car trips represent 89.5%, transit 4.3% and walking trips 6.2% of total trips. Car trips are shorter than public transit trips both in distance and duration and individuals are, on average, in higher income bracket (5.61 v. 5.07). However, public transit trips are more common in denser areas of the city (6.28 v. 4.88) The average public transit trip involves 2.28 transfers or change of mode while car and transit trips do not involve any transfer (The number of transfers is the number of trip legs minus 1). Finally, walking trips are shorter, about 11 minuts and half a mile, and made by commuters in lower income brackets and relatively denser areas.

Table 2.1: Choice Data: Descriptive Statistics

	Full Sample		
	Car	Public Transit	Walk
duration (min.)	24.70	49.57	11.73
distance (miles)	11.94	15.76	0.55
legs (#)	1	3.28	1
mean income (bracket)	5.61	5.07	4.65
density origin (bracket)	4.88	6.28	5.47
number of observations	24.721	1.200	1.715

The main trip characteristic for which I do not have data is trip money cost for each mode. Hence, I have to simulate trip costs. To recover car trip cost I use trip distance, $d^c_{i(o,d)}$, and vehicle type, v^c_i, which I observe from the CHTS together with the American Automobile Association \$/mile estimates by vehicle type, $dollars(v^c_i)$. Hence, I define car-trip cost as:

$$p^c_{(o,d)} = d^c_{(o,d)} dollars(v^c_i)$$

Next, public transit trips are defined by the Metro LA 2012 fare schedule. Hence, price is just a function of trip duration, $t^b_{i(o,d)}$, and starting time of the trip, $st^b_{i(o,d)}$:

$$p^b_{i(od)} = f(d^b_{i(o,d)}, st^b_{i(o,d)})$$

Finally, walking trips are free of charge and I set $p^w_{i(od)} = 0$. This completes all relevant trip characteristic variables for realized trips.

However, the CHTS does not provide trip characteristics for non realized trips which are fundamental to estimate the demand model. That is, I need to know which were the trip characteristics of non chosen alternatives when the commuter made her decision. To overcome this problem I augment the data set with trip characteristics for unobserved modes. Crucially, travel diaries provide trip origin and destination tracts, departure time, and day of the week when the trip was realized. With this information I use Google maps' directions API to request trip characteristics for a counterfactual trip on the same day of the week, at the same time of the day and from origin to destination tract centroids. This allows me to obtain trip characteristics from Google maps both for the observed and unobserved modes.[2].

Finally I validate Google's response against the CHTS for the trips with observed mode choices. To do so I regress the survey value against Google's answer for the observed modes:

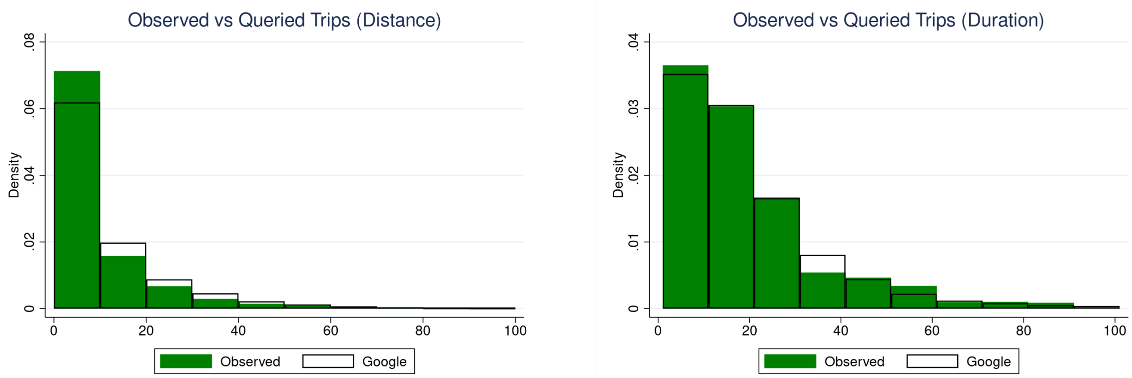
[2]Application Programming Interface, Directions API here

$$log(y_i^{survey}) = \beta_0 + \beta_1 log(x_i^{google}) + \epsilon_i$$

The parameter of interest is β_1. If this value is close to 1 the survey and Google tend to give the same answer and then I can confidently use Google's response for not observed mode trip characteristics in the estimation procedure. Table (2.2) shows the results of these regressions. Survey and Google answers are very similar in terms of distance both for car and public travel modes. Google is still very similar to survey answers in terms of time for car commuting. The value of the parameter for bus time is 0.7, which is not as close to 1 as the others. This may be due to the fact that the public transit network has changed in terms of bus/train frequencies and schedules. However, the value is still close to 1. Moreover, in figure (2.2) I show density plots of observed (from the survey) and Google answers for bus trip distance and duration. The distributions are pretty similar which makes me confident of using Google's answers as characteristics for counterfactual trips.

	Car		Bus	
	Distance	Time	Distance	Time
β_1	0.99	0.89	0.95	0.70
R^2	0.93	0.70	0.90	0.65

Table 2.2: Survey v. Google Answers



(a) Trip Distance

(b) Trip Duration

Figure 2.2: Density Plots of Survey and Google Answers for Bus Trips

2.2.2 Demand Estimation and Results

I bring different versions of equation (2.1) to the data:

$$u_{i(o,d)}^{jr} = X_i\alpha^j + \beta^j + \beta_t t_{(o,d)}^j + \beta_p p_{(o,d)}^j + \epsilon_{ij}$$

Table (2.3) reports parameter estimates. All sign parameters are in the right direction, all are negative, meaning that trip travel time and travel money cost generate disutility to the commuter. Moreover, $\hat{\beta}_p$ is around 3 times higher in absolute value than the travel time counterpart in column (2). This relationship is stable to the inclusion of different sets of controls. Column (3) includes individual income as control, column (4) adds departure location population density and finally, column (5) adds other individual characteristics such as gender, age or education level. However, in the simulation of the model I will use parameter estimates from column (2) since I don't include individual or location characteristics at this stage.

With the above parameter estimates I can compute the value of time (VoT) for each

	(1)	(2)	(3) Inc	(4) Inc, Dens	(5) Inc, Dens Individual
β_t	-0.046^{***}	-0.070^{***}	-0.079^{***}	-0.074^{***}	-0.067^{***}
	(0.005)	(0.008)	(0.009)	(0.009)	(0.009)
β_p		-0.212^{***}	-0.227^{***}	-0.254^{***}	-0.262^{***}
		(0.033)	(0.032)	(0.033)	(0.036)
Base Alternative			walk		
VoT ($/hour)		19.81	20.88	17.48	15.34
Log-Likelihood	-238.84	-207.07	-197.38	-174.62	-171.76

Table 2.3: Demand Estimation Results

specification. I use the following formula to recover the VoT[3]:

$$VoT = \left(0.6\frac{\beta_t}{\beta_p}\right)100$$

Taking into account that the median $20.52 hourly wage in the Los Angeles County is $20.52 per hour, I'm confident that the above estimates are within a reasonable accuracy margin[4].

2.3 Supply Estimation

As seen in the previous chapter, an essential component of the supply side of the model is the link congestion function. This function captures the relationship between traffic flow in a link and the speed/travel time in that link. The link capacity function is a critical component of the model because it is this function that translates increases in car flows to increases in travel times. Without an accurate link capacity function, it's impossible to accurately model the user's route choice behavior, which is based on a commuter's perception of travel cost.

[3]Frank S. Koppelman, Chandra Bhat A self instructing course in mode choice modeling; multinomial & nested logit models U.S. Department of transportation, Federal Transit Administration (2006) and Estimation of Value of Travel Time for Work Trips, Athira, Muneera, Krishnamurthy, Anjaneyulu (2016)

[4]Bureau of Labor Statistics

In 1964 the US Bureau of Public Roads (BPR) developed the most widely used of such functions. Using the notation from the previous chapter (1), this function is:

$$t_l = t_l^0\Big(1 + \alpha(\frac{m_l}{c_l})^{\beta}\Big) \quad \forall l \in \mathcal{L}^R \tag{2.2}$$

where t_l^0 is free-flow travel time in road network's link l and m_l and c_l are flow and capacity on that link respectively. For ease of notation I suppress link's subscript. Let the ratio t/t_0 be the travel time multiplier. Parameters α and β need to be estimated.

Note, that as required by the model, the BPR function is nonnegative, single-valued, monotonically increasing and strictly convex. Moreover, note that if no car is in the link ($m = 0$) the amount of time to traverse the link is equal to the free flow time t^0. Travel time starts to increase as m increases and when the ratio $m/c > 1$ travel time increases exponentially. Figure (2.3) shows how travel times increase (blue line) as the volume/capacity ratio increases from 0 to 1.5.

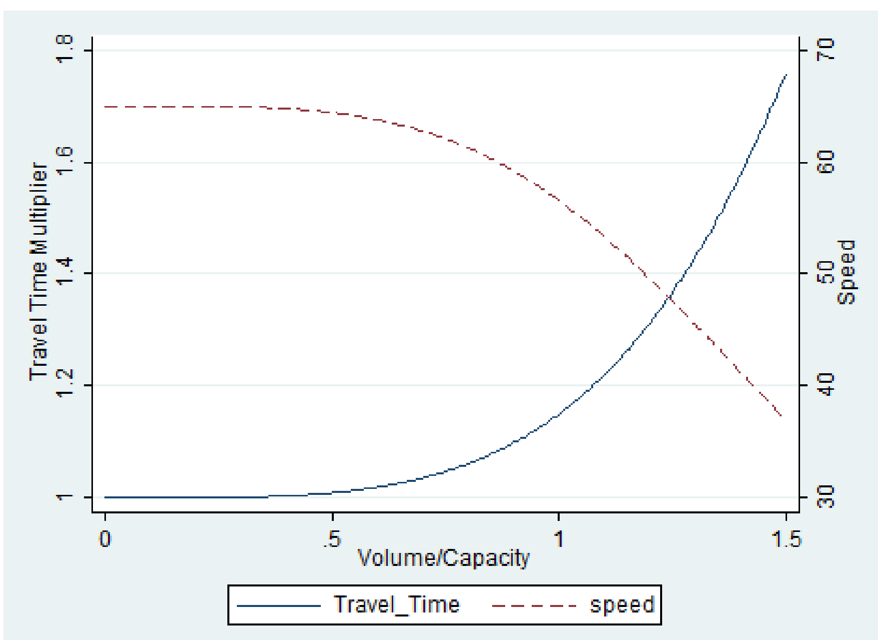


Figure 2.3: BPR function with $\alpha = 0.15$ and $\beta = 4$

The BPR also proposed the most widely used calibration of parameters $\alpha = 0.15$ and $\beta = 4$. Multiple studies have used the calibration of such function given by the BPR. However, almost 60 years have passed since that calibration was proposed. In that time cars and highways have evolved making previous estimates inaccurate. Moreover, different areas have different demographics, economic, cultural and behavioral characteristics that can affect the relationship between flow and travel times.

To actualize the value of this parameters and adapt them to the particular environment of the Los Angeles county I present an estimation technique and data that allows to estimate them in the next subsection. Moreover, I use the example of the Sioux Falls network, introduced in the previous chapter, to illustrate how results change when using the estimated parameters or the parameter values proposed by the BPR.

2.3.1 Data, Estimation and Results

The data used to estimate the parameters of the Los Angeles' BPR function comes from the Caltrans Performance Measurement System (PeMS). All major divided freeways in California contain embedded loop detectors that continually measure the number of vehicles crossing the detector and the average time that each vehicle spends over the detector. Using these data, PeMS constructs 5-minutes and hourly measures of vehicle flows and average vehicle speed for each detector. The sample at hand consists of 4.355 detectors observed during January 2012 in Los Angeles county (see figure (2.4)). This adds up to 3.118.929 detector-hour observations. The average loop detector covers 0.68 miles and 3 line segments. The free flow speed on the county is 65 miles per hour.

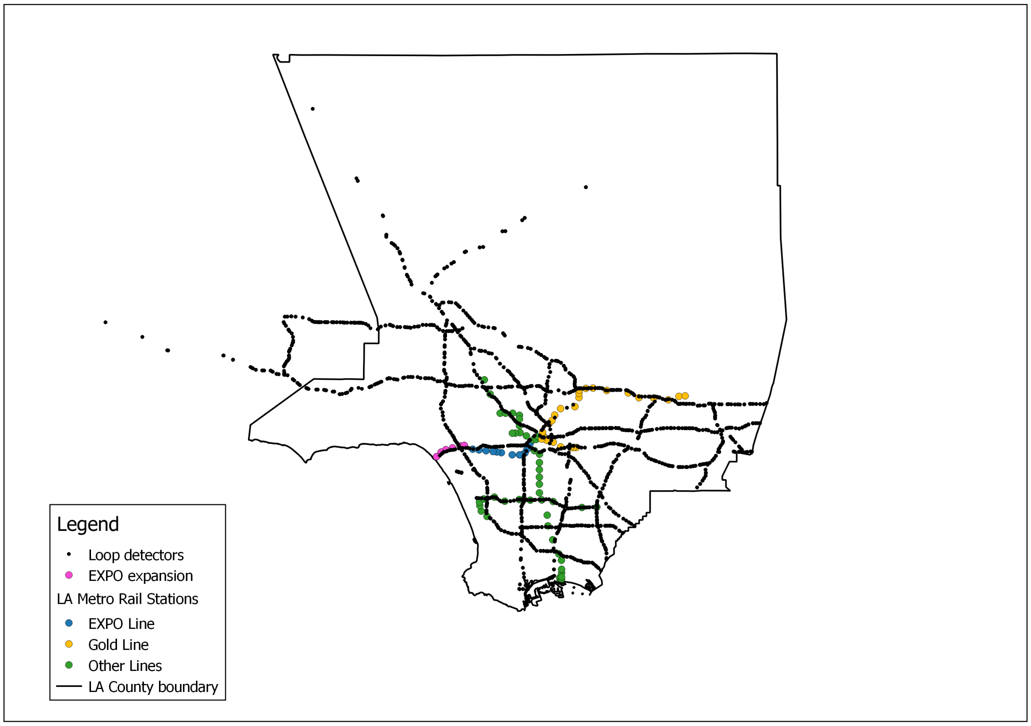


Figure 2.4: Loop Detectors in Los Angeles County

Table (2.4) presents average lane occupancy and speed and total flow in the loop detectors in the LA County highway system. I separate the sample into rush and not rush hours to highlight the difference in network use[5]. As expected, average occupancy and flow of cars is higher during rush hours and average speed is far from the free flow speed. During off peak periods average occupancy and total flow decrease and speed (63.5 miles per hour) gets closer to the free flow speed.

	Hour		Difference	
	Rush	Not Rush	Total	Percent
Avg Occupancy	.097	.052	0.045	55.25
Total Flow	3624.64	2334.72	1289.92	86.37
Avg Speed	57.63	63.34	-5.79	-9.01

Table 2.4: PeMS Data Summary Statistics

Finally, free flow speeds, t^0, and capacity, c, are not observed directly from the data.

[5]Rush hours are defined by Metro LA as weekdays from 6am to 10am and from 3pm to 7pm.

However, for each detector i in the sample I do observe the average length covered by the detector, $length_i$, posted speed on that segment s_i, and the number of lanes covered, $lanes_i$. With this I define t_i^0 and c_i as[6]

$$t_i^0 = \frac{length_i}{s_i} \quad \text{and} \quad c_i = 200 \times lanes_i \quad \forall i \tag{2.3}$$

This data allows me to plot the relationship between speed, s, and flow, m, in a road segment for each hour interval (figure 2.5). This scatter plot should represent the inverse relationship expressed by equation (2.2)[7]. Yet, the graph shows that not only are the plot shapes different, but there is no unique functional relation between speed and flow. This contradicts the assumptions of the link capacity function, namely, that travel time is a continuous, monotonic, strictly increasing and strictly convex function of the flow. Hence, estimating the parameters of (2.2) with the data at hand will lead to misleading results.

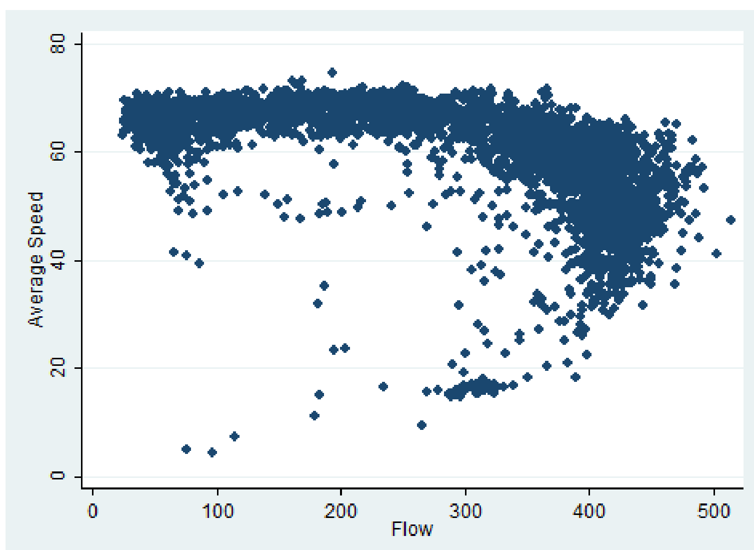


Figure 2.5: Empirical Speed-Flow relationship. Loop detector id=715898

[6]check this for capacity link

[7]Note that, for a 1 mile segment, time is the inverse of speed: $t = 1/s$.

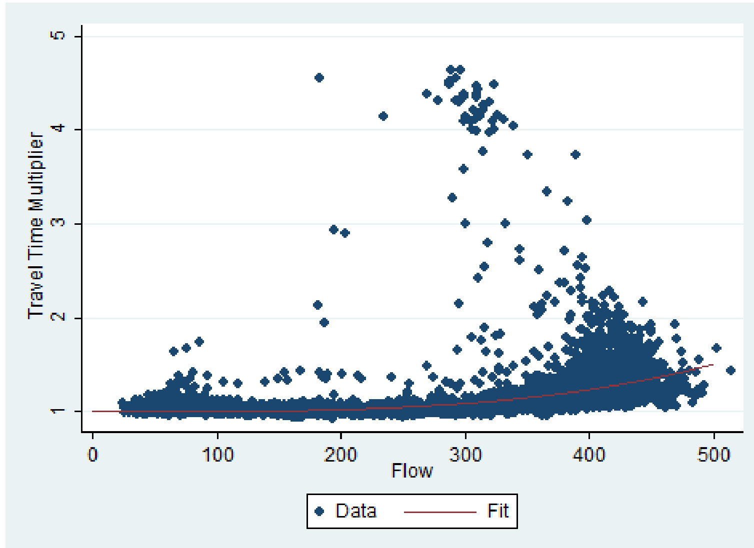


Figure 2.6: Empirical Travel Time-Flow relationship and Fit. Loop detector id=715898

To overcome the previous problem I can extend the measured speed and flow with quasi-density (k) using the fundamental diagram of traffic congestion. The fundamental diagram of traffic flow captures the relation between speed, flow and density. This relationship is described with the equation:

$$m = sk = \frac{miles}{time}\frac{cars}{miles} = \frac{cars}{time} \tag{2.4}$$

Panel A of figure (2.7) represents the speed-flow curve, that is the theoretical counterpart of figure (2.5). Trying to estimate this relationship would lead to inaccurate values due to the fact that for some flow values, average speed can take multiple values. Using equation (2.4) I can obtain the relationship between speed and density as in panel B of figure (2.7). Estimating this relationship will lead to more accurate parameter values. The empirical relationship between speed and density is in figure (2.8) and the time counterpart in figure (2.9).

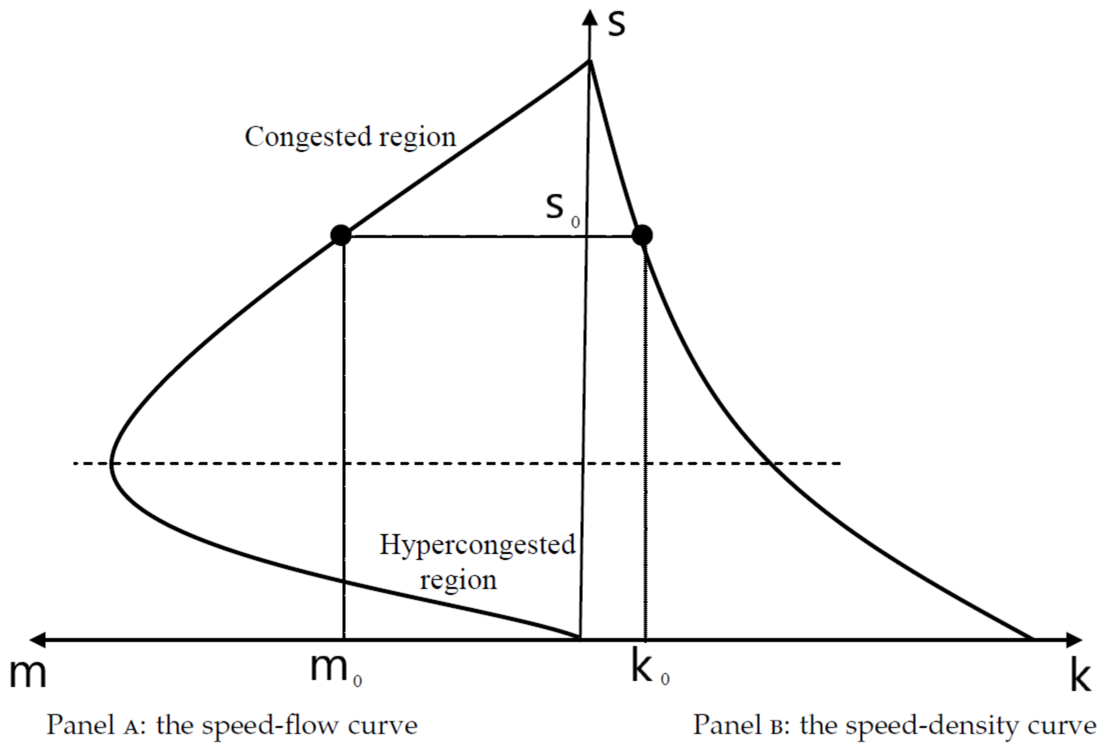


Figure 2.7: Fundamental Diagram of Traffic Flow

Interestingly, both resemble now the typical BPR function where speed (time) decreases (increases) steadily until the capacity threshold is reached and starts to fall (rise) sharply when capacity is exceeded. This was not possible for the empirical flow-speed relationship which did not have a unique functional form at all. While the flow-speed relation does not have a unique functional form, the density speed relation does have one: that is, for a given density rate we can estimate a unique flow-speed value. This finding will be used to reformulate the BPR functions.

Now that I have shown that the BPR function shape is observed against density and not a flow, I have to obtain densities in the assignment model where they are not available. To this end, let me further exploit the discrepancy between observed flow and the flow of an assignment model, which is not constrained by the capacity of the road. Measured flow is strictly constrained by the traffic flow dynamics and any observation contradicting them might only result from measurement errors. Consequently, the measured flow cannot exceed capacity and drops down when the demand volume exceeds the maximal density (figure). On the other hand, the flow coming from an assignment model is constrained only by the assumptions of the traffic flow model, which is by itself a significant simplification. The flow from the assignment model may exceed capacity and it gets severely delayed but in general is allowed. The macroscopic flow is in principle proportional to the demand (the higher the demand the higher the flow) while the observed physical flow is different (it grows with demand up to capacity and falls down afterwards).

I follow Kucharski and Drabicki (2017) ([**?**]) in using the mapping from observed quasi-densities to flows of the assignment problem:

$$m = c\frac{k}{k(c)} \tag{2.5}$$

The inverse of this mapping allows me to express quasi-density as a function of the assignment flow, m, and the density-at-capacity to capacity ratio, which is a constant:

$$k = m\frac{k(c)}{c} \tag{2.6}$$

Given that I've shown that the BPR function can be empirically observed in terms of density and not of flow, I can use (2.5) into the BPR function (2.2) to obtain the travel time multiplier as a function of density:

$$\frac{t}{t_0} = 1 + \alpha\Big(\frac{k}{k_c)}\Big)^{\beta} \tag{2.7}$$

Since travel time multiplier is not observed I need to further transform the above equation to use it in the estimation problem. To do so, recall that:

$$\frac{t}{t_0} = \frac{s_0}{s} \tag{2.8}$$

Therefore, the model theoretical speed is

$$\hat{s} = \frac{s_0}{1 + \alpha(k/k_c)^\beta} \tag{2.9}$$

which is a function of the observed physical flow and can be compared with the observed speed in the estimation problem: match the measured speeds with the modeled speeds to minimize the sum of square residuals.

$$\min_{\alpha,\beta} \sum_{it} \Big(s_{it} - \hat{s}_{it} \Big)^2$$

Table (2.5) presents parameter estimates when using flow (column 1) and density (column 2) as explanatory variables. As expected, the goodness of fit (measured as the R^2) increases from 22% to 64%, a considerable increase. The parameter estimate for α when using density is 0.295 which is almost double than the one proposed by the BPR. The main difference between the two specifications is that $\hat{\beta}_{density}$ is 3.429 which is 0.7 higher than $\hat{\beta}_{flow}$

Variable	(1) Flow	(2) Density
α	0.254***	0.295***
	0.00023	0.00015
β	2.714***	3.429***
	0.00425	0.00144
Observations	8,051,035	8,051,035
R-squared	0.220	0.638

Standard errors in parentheses
$^{***}p < 0.01, ^{**}p < 0.05, ^{*}p < 0.1$

Table 2.5: Congestion Function Estimation Results

Finally, figure (2.10) plots the congestion function ((2.2)) under the BPR parameters and the ones estimated above. In the estimated function (blue) congestion kicks in at lower values of volume/capacity than in the benchmark BPR. This is due, mainly, to the fact that $\hat{\alpha} > 0.15$. Hence, the Los Angeles County highway network gets congested for lower levels of the volume/capacity ratio than what is predicted by the BPR.

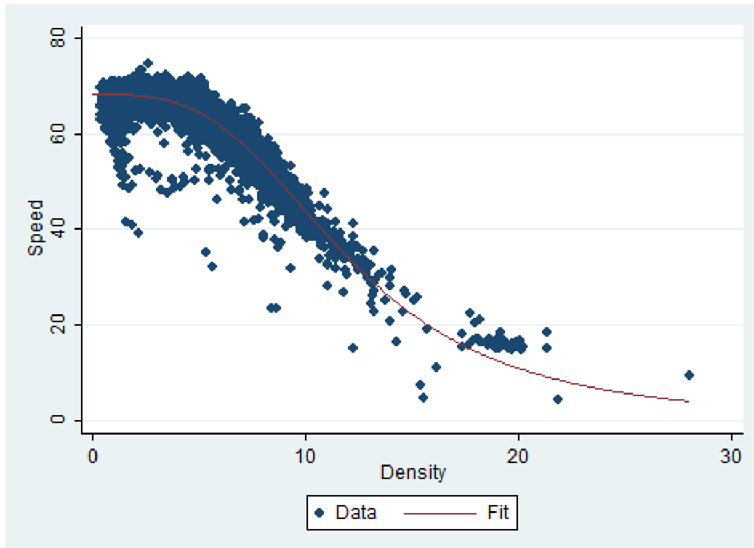


Figure 2.8: Empirical Speed-Density relationship and Fit. Loop detector id=715898

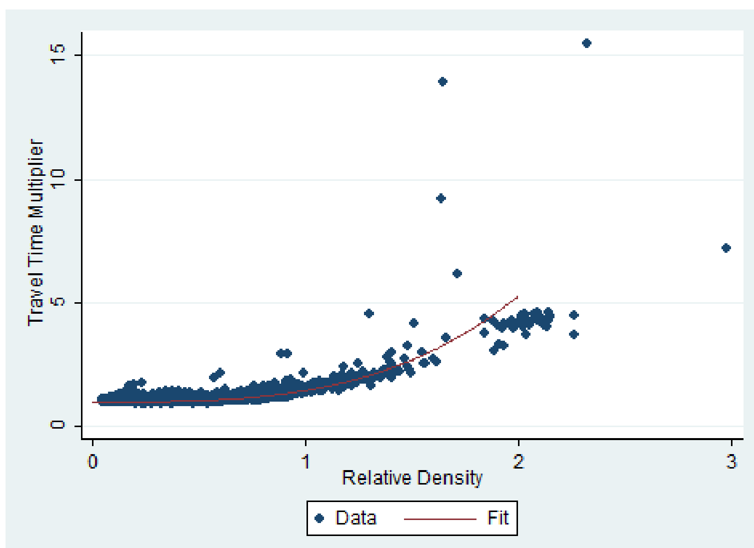


Figure 2.9: Empirical Travel Time Multiplier-Density relationship and Fit. Loop detector id=715898

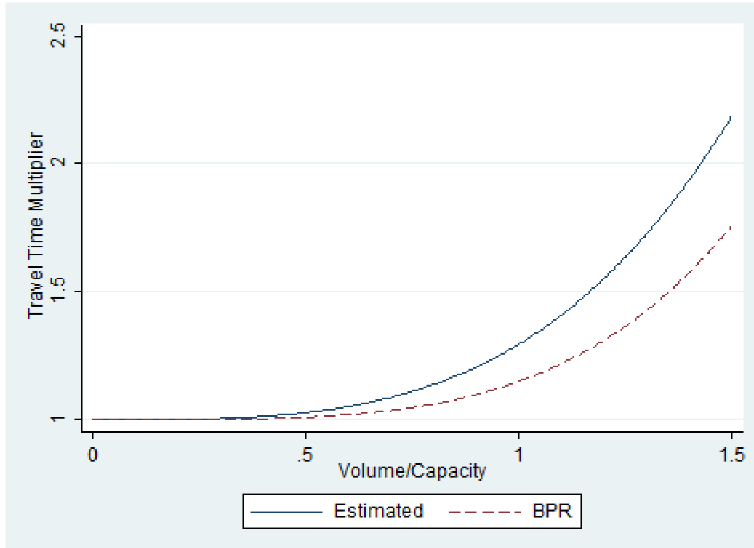


Figure 2.10: Estimated congestion function for L.A. vs. BPR with ($\alpha = 0.15$, $\beta = 4$)

2.3.2 Application

To demonstrate how the results are affected by the parameters α and β I solve the traffic assignment problem of the city of Sioux Falls, South Dakota. The Sioux Falls problem is used extensively as a benchmark in the Civil Literature engineering due to its simplicity.

The Sioux Falls road network is composed of 24 nodes and 76 links (see figure (2.11)). Each link has attached a BPR congestion function with parameters $\alpha = 0.15$ and $\beta = 4$. Links differ in length and capacity. Each of the 24 network nodes is at the same time an origin and a destination node. The origin-destination demand matrix is in 2.12. The problem consists on assigning this demand to the network and to recover equilibrium travel times.

In the original Sioux Falls network all links are of the same type. To introduce the distinction into street segments and highway segments I randomly assign 17 links ($\sim$22.3%) to be highway links and change the value of the BPR's parameters by the ones estimated in the previous section. Finally I compare the equilibrium resulting in both networks. (See table (2.11)).

Converting 22% of the links into highway links changes travel times by more than 1% in about 80% of the links and by more than 5% in 50% of the links. As extreme cases, around 13% of links change their travel time by more than 20%.

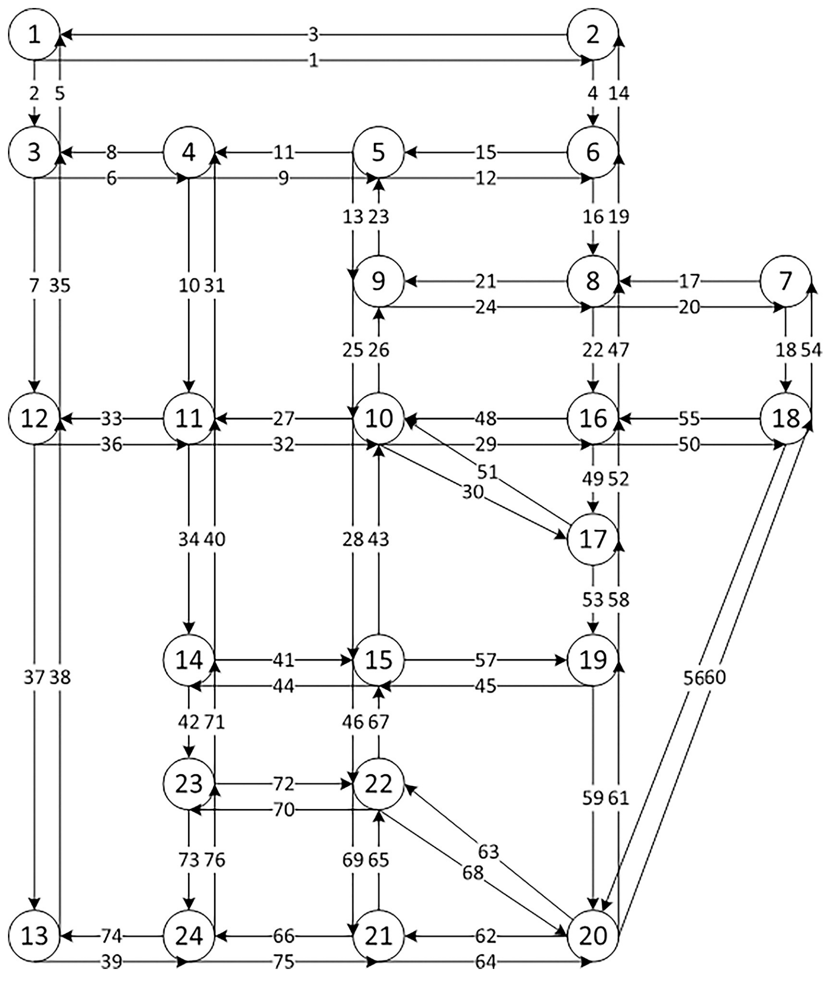


Figure 2.11: Sioux Falls, SD, Road Network

2.4 Model Evaluation and Counterfactual Data

Model evaluation and counterfactuals revolve around the typical weekday 8 a.m. commute. Here I present the data needed to simulate this commute and to run the model.

There are two main data sets needed: commute/demand data and transportation network data.

2.4.1 Morning Commute Data

To simulate a typical 8am morning commute in Los Angeles County I use the Longitudinal Employer-Household Dynamics (LEHD) Origin-Destination Employment Statistics (LODES) for 2016 to simulate home tract-to-work zip commute flows. An advantage of the LODES data set is that it keeps a historical record of commute flows since 2002. I will use the year 2012 data to evaluate the model's performance against observed data in the next section.

LODES data reports total flow of workers on a typical day from census block to census block. Two problems arise: i) census block is too granular and running the model at this level of spatial disaggregation would be unfeasible, and ii) flows reported by LODES are total day flows and are not disaggregated by time blocks.

To solve the first problem I define residential locations to be US Census tracts and working locations to be ZIP codes. There is a well defined many-to-one mapping between census blocks and census tracts that I use to aggregate them. However, there is no clear mapping between census blocks and ZIP codes. I overlay census blocks over ZIP codes using GIS and assign the fraction of destination commuters that corresponds to the fraction of land inside the ZIP code.

To rescale the flows by departure time I use the American Community Survey (ACS) departures by 30 minute time bracket estimates. The ACS reports the share of commuters departing by origin location not for bilateral flows. Hence, the assumption is that within a census tract and at given 30 minute bracket the share of commuters departing to a destination is the same as the share of total commuters going to that destination during the day.

Formally, let M^{total} be a matrix of bilateral flows coming from LODES data, and s_{8am} be a column vector with entries the share of departures by origin at 8:00am from the ACS. Hence, the commuting origin-destination demand matrix used in the simulation is:

$$M = s^{8am} \otimes M^{total} = \begin{bmatrix} s_1^{8am} M_{11}^{total} & s_1^{8am} M_{12}^{total} & s_1^{8am} M_{13}^{total} & \dots \\ \vdots & \vdots & \vdots & \dots \end{bmatrix}$$

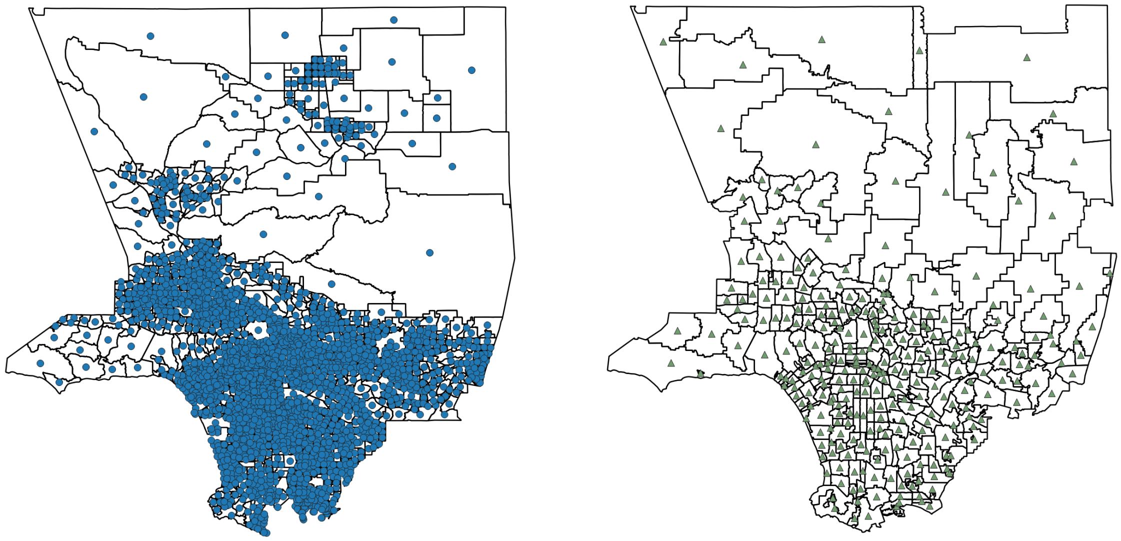
Therefore, all simulations of the model will be at 8am at try to replicate the commuting market at 30 minutes blocks.

2.4.2 Transportation Network Data

The road network data comes from the Southern California Association of Governments (SCAG). The final network is composed of more than 30.000 links representing segments of roads[8]. I add additional links connecting location centroids to the network. For each link I observe road type, length, posted speed, and number of lanes. With this information I can obtain link capacity and free flow travel time for all links.

Finally, the network consists of 1.623 centroids, 1.443 census tracts that are origin locations and 180 zip codes that are destination locations (residential locations are dots (panel a) and working locations are triangles (panel b) in figure (2.12)). Hence there are 1.623 connector links from centroids to the road network. The road network has 29.347 links and 16.247 nodes. See the final road network in figure (2.13).

[8]The original network covers Southern California minus the county of San Diego and is composed of more than 100,000 links. I trim the original network to get only the Los Angeles County network. I further reduce the network by means of an iterative procedure: I perform car traffic assignment and save all the links that get some traffic. I remove these links from the network, making sure that the networks is still fully connected, and perform car traffic assignment again. I delete all links with no flow after these two iterations. I have to do this due to the computational burden that imposes working with the entire network.



(a) Residential Locations

(b) Working Locations

Figure 2.12: Residential and Working Centroids Location

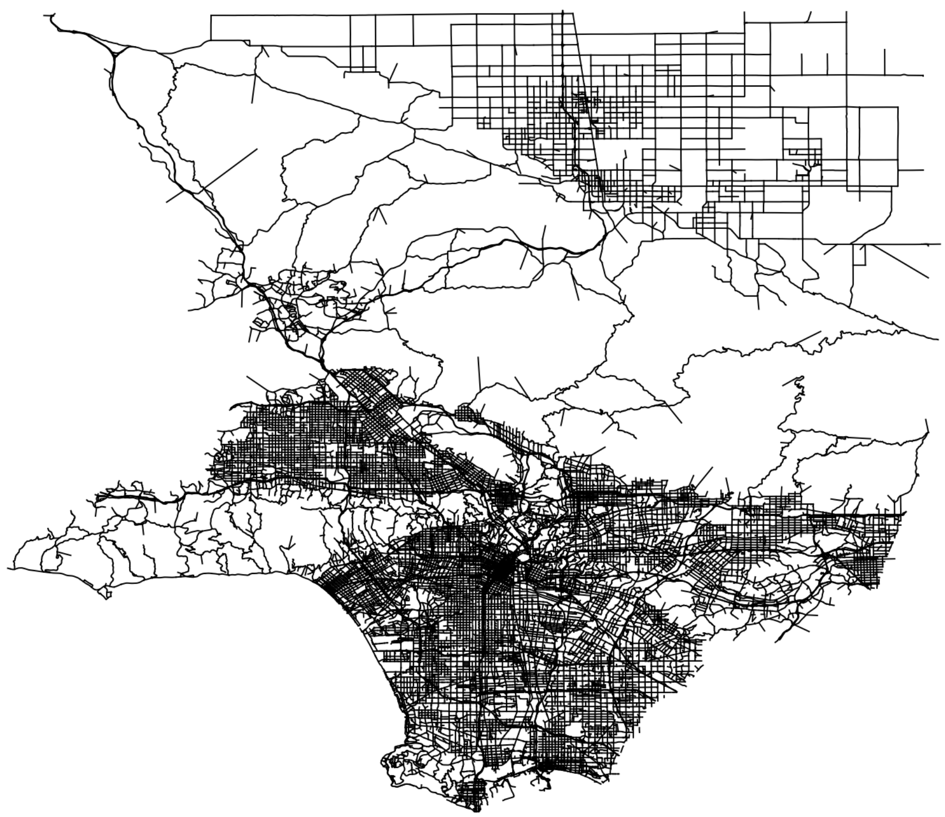


Figure 2.13: Los Angeles County Road Network

To obtain travel times and monetary cost for the public transit and walking network I use Open Trip Planner (OTP). Open Trip Planner provides directions similar to Google Maps and trips can be planned around an arbitrary public transit schedule. Since I am simulating an historical event I use 2016 public transit schedule. In 2016 the public transit network on Los Angeles county is composed of 140 bus lines with more than 7.000 stops and 6 rail lines with 93 stops (see figure (2.14) to see the rail system in Los Angeles county).

One advantage of OTP is that works with General Transit Feed Specification (GTFS) data. GTFS "defines a common format for public transportation schedules and associated geographic information. GTFS "feeds" let public transit agencies publish their transit data and developers write applications that consume that data in an interoperable way"[9]. In particular, the GTFS allows to implement modifications in the public transit network and recalculate all trip characteristics for public transit. Therefore, in each of the counterfactuals, I will simulate a change in the public transit system by changing the 2016 MetroLA's GTFS schedule, feeding it to the OPT simulator, and recalculating all public transit travel times and monetary cost[10].

When simulating a bimodal public transit system (rail and bus) the GTFS file is divided into two subfiles, one that contains all information of the bus system and another that contains rail information. Each file can be modified independently. Moreover, there are repositories that keep historical GTFS records such as transitfeeds.com [11]. The oldest GTFS bus file is from year 2013 while rail file is from 2016.

[9]For more information see google's gtfs page

[10]Check Metro LAs gtfs schedules for more information

[11]See transitfeeds.com for LAC's historical data.

2.5 Model Evaluation

Before running the different counterfactuals I compare the model's performance against Los Angeles County commuting market observed data. If the model is able to replicate the main moments of the data, when I introduce modifications in the network I can attribute any change in these moments to the changes in the network.

Since the data that I observe comes from the CHTS-2012 I modify the 2016 rail GTFS file to replicate the 2012 rail system and use the 2013 bus GTFS. I feed these two files into OPT and calculate all bilateral public transit commute travel times and monetary costs.

Utilizing parameter estimates for demand and supply from the previous sections, the 2012 public transit network, and 2012 commute flows, I simulate the 8am commute of a typical weekday and compare it to the data from the CHTS-2012.

Table (2.6) presents the results of this exercise. Non of the moments in the data is targeted when running the model. The model captures accurate travel times for the three modes available and mean travel time. However, the model makes more attractive public transit than commuting by car and so public transit share is higher in the model than in the data and the opposite is true for car commuting. Despite the good fit of the model, I'm confident that with some fine tuning and the inclusion of origin tract characteristics, such as population density, this gap between the model generated data and the observed data can be further reduced.

	Model	Data
Mean car travel time (min.)	26.61	28.55
Mean bus travel time (min.)	46.82	48
Mean walk travel time (min.)	26.69	25
Mean car share (%)	78.69	84.79
Mean bus share (%)	16.06	11.23
Mean walk share (%)	5.22	3.97
Mean travel time (min.)	29.85	31.3

Table 2.6: Los Angeles County Commuting Market: Model v. Data

2.6 The Impact of the Expo Line Extension in Los Angeles

In this section I use the model to evaluate the impact of the extension of the Meto LA's expo line. The Expo line (light and dark blue lines in figure 2.14) connects downtown Santa Monica to downtown Los Angeles by means of 15.2 miles and 19 stations. The line was open to the public in two phases: the first phase connected downtown Los Angeles to Culver city and was opened in 2012 (light blue line). The second phase extended the line to Santa Monica adding 6.6 miles and 7 new stations in 2016 (dark blue line). The final cost of Phase 1 was $979M while for the second phase was $1.511B. The total cost of the line is of $2.49B or $160M per mile. As of 2018 it has an average weekday ridership of 61,957 persons.

In this exercise I focus on phase 2 (the extension) of the Expo line. To have a sense of how this extension changed public transit travel times, Figure (2.15) shows public transit commuting isochornes from downtown Santa Monica before (panel a) and after (panel b) the Expo line extension. After the expansion, the area reachable by public transit within 90 minutes is considerably extended, specially towards south Los Angeles. Commuters from Santa Monica can reach now downtonw Los Angeles faster and from here redirect to other areas of the city. Figure (2.16) repeats the same exercise but starting from downtown Los Angeles. In this case, the change is not that important and the only gain is towards the Santa Monica area, the rest of the isochrones are unchanged.

The strategy I use to evaluate the commuting and welfare impact of the Expo Line extension is the following: first, I modify the GTFS file to incorporate the stations and time schedule of the new Metro Light Rail system. Then I recalculate all origin-destination commute times and cost under the new public transit network. This extension implies the change of 16,358 origin-destination public transit travel time commutes, 6.29% of all 259,740 total origin-destination commutes. Next, I solve the model using these new public transit network and compare this equilibrium to the benchmark scenario (no extension)[12].

Table (2.7) reports the results of this exercise. Column (1) shows the results before the expansion while column (2) shows the results after the expansion and in the presence of congestion. The extension implies that in the new equilibrium 88.79% of commutes are affected, that is, the change in the original 16k commutes translate into a change in more than 230k commutes because of the congestion externalities that trigger changes in routing and mode choices.

Column (3) in table (2.7) shows the results of simulating the Expo line extension without accounting for congestion. As explained before, the lack of congestion externalities does not trigger the travel cost - optimizing choices in the model and just the 16,358 origin-destination pairs affected by changes in travel times are affected in equilibrium. Car share (public transit share) is lower (higher) than in the presence

[12]For computational reasons I simulate the southern most part of the Los Angeles County. Mode shares are a bit different from those in table (2.6) because here I'm considering the most dense area of the county and therefore bus share is a bit higher and car share is a bit lower.

of congestion externalities. In this new setting the increase in welfare is of 0.198%, 2.33 times higher that in the presence of congestion externalities.

Next I compute the time to recover the investment in each scenario: with and without congestion externalities. To do so, I make the following three assumptions: first, using the estimates of column (2) of table (2.3) I find that the VoT is 19.81$/hour, second, the road network is congested during 8 to 16 hours during a typical weekday (see figure (2.26)), and third, the network is not congested during weekends (see figure (2.27)).

Notice that the simulation is at 30 minutes block level, hence, to obtain the yearly dollar impact of the Expansion I compute:

$$V_{extension} = \underbrace{(2 * 16 * savings)}_{1} * \underbrace{260}_{2} * \underbrace{VoT}_{3}$$

Where the first term converts hours saved by 30 minute blocks into hours saved daily, the second term multiplies by working days per year, and the last term is the value of time computed in section 2.2. where I find it to be 19.81$/hour.

I find that the value of yearly hours saved by the expansion is $235,294,241.72 in the presence of congestion and $561,374,195.2 in the no congestion case. In the case where we don't have into account congestion externalities, as expected, we are overestimating the value of the extension with respect to the case with congestion. In particular, I find that the no congestion value of the extension is 138% higher to the case with congestion. With that I find that to recover the investment in the presence of congestion externalities is of 6.37 years and 2.67 years when congestion is not present.

Table 2.7: Expo Line Extension Counterfactual Results

	Before Expansion (1)	After Expansion (2)	(3)
Congestion Ext.	Yes	Yes	No
Car Share	77.95	77.31	77.25
Bus Share	17.23	17.91	17.96
Walk Share	4.82	4.78	4.79
Commutes Affected	.	230,647	16,358
Commutes Affected (%)	.	88.79	6.29
Total Time (hours)	599,126.13	597,698.54	595,719.23
Welfare	-1,786,697.15	-1,785,171.69	-1,783,164.58
Welfare Change (%)	.	0.085	0.198

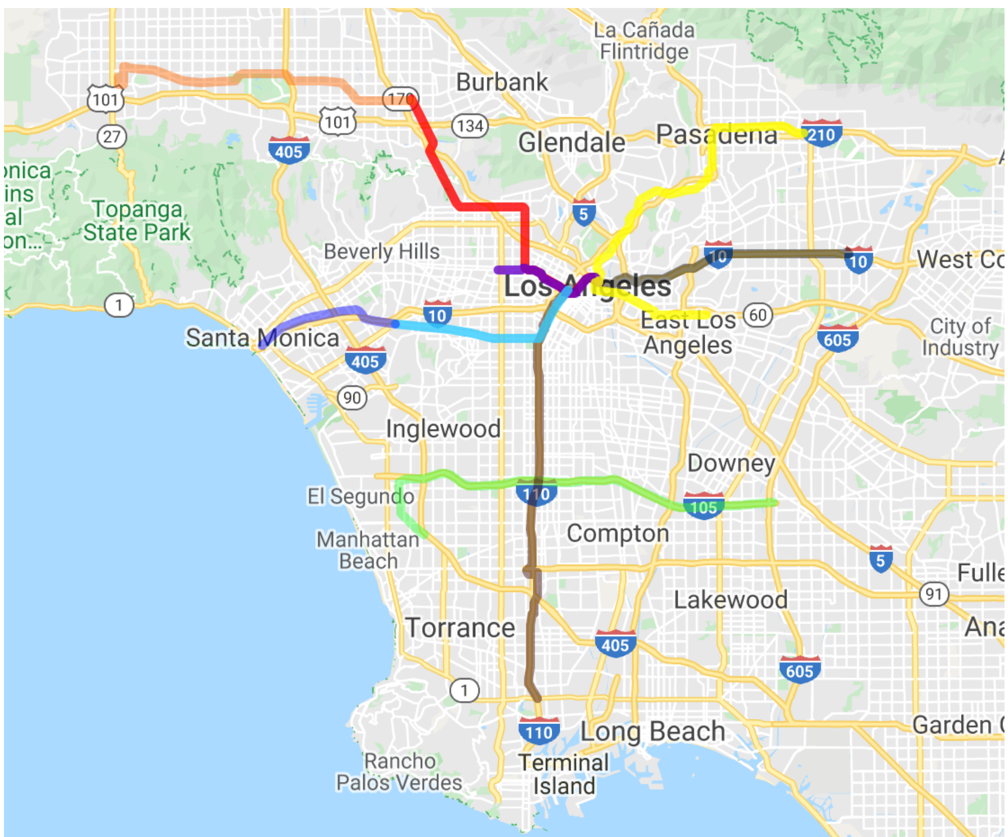
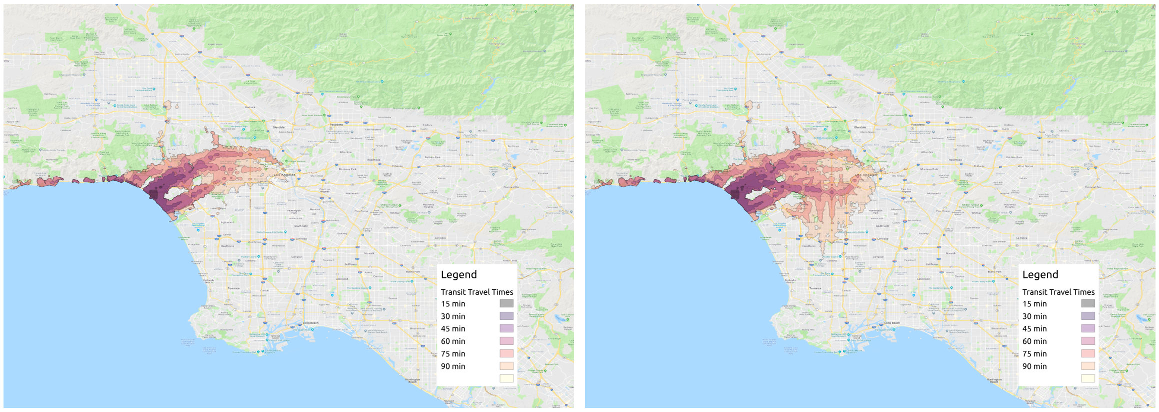


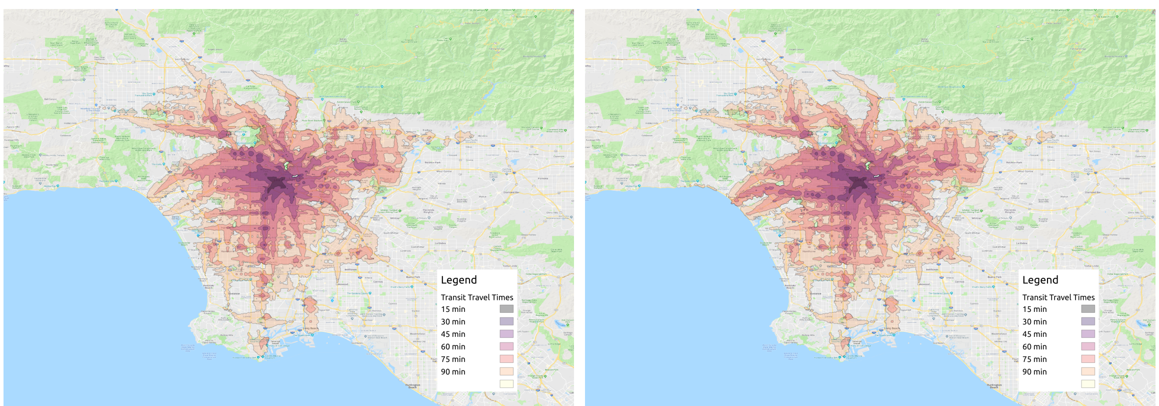
Figure 2.14: Los Angeles County Light Rail Network

Figure 2.15: Isochrones From Downtown Santa Monica



(a) Isochrones without Expo Line Extension (b) Isochrones with Expo Line Extension

Figure 2.16: Isochrones From Downtown Los Angeles



(a) Isochrones without Expo Line Extension (b) Isochrones with Expo Line Extension

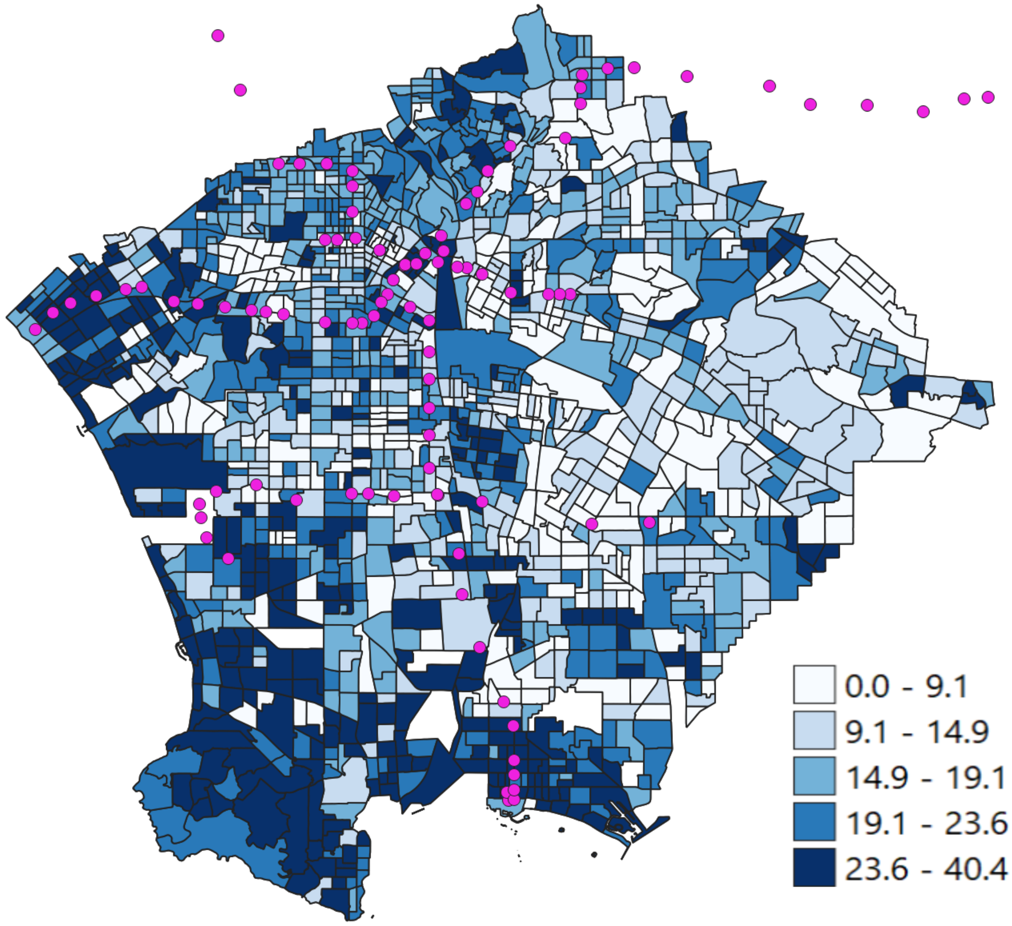


Figure 2.17: Southern Los Angeles County Public Transit Share by Census Tract Before Expo Line Extension

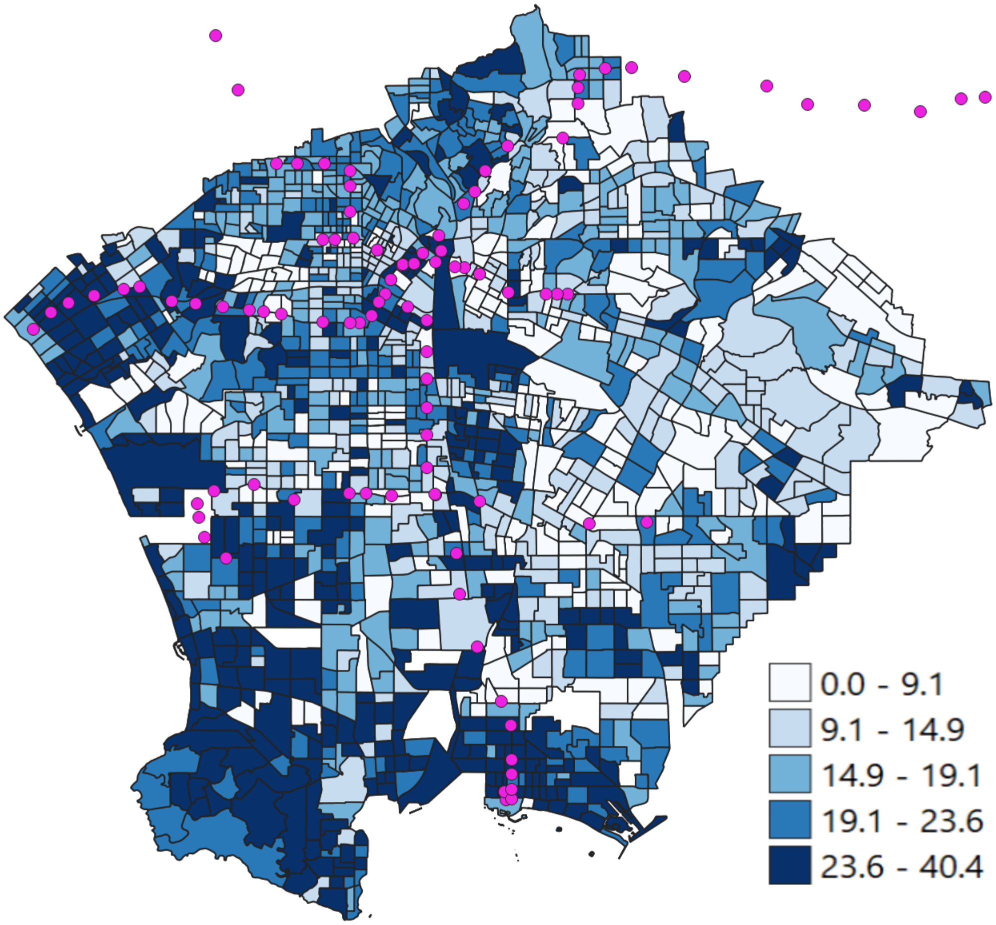


Figure 2.18: Southern Los Angeles County Public Transit Share by Census Tract After Expo Line Extension

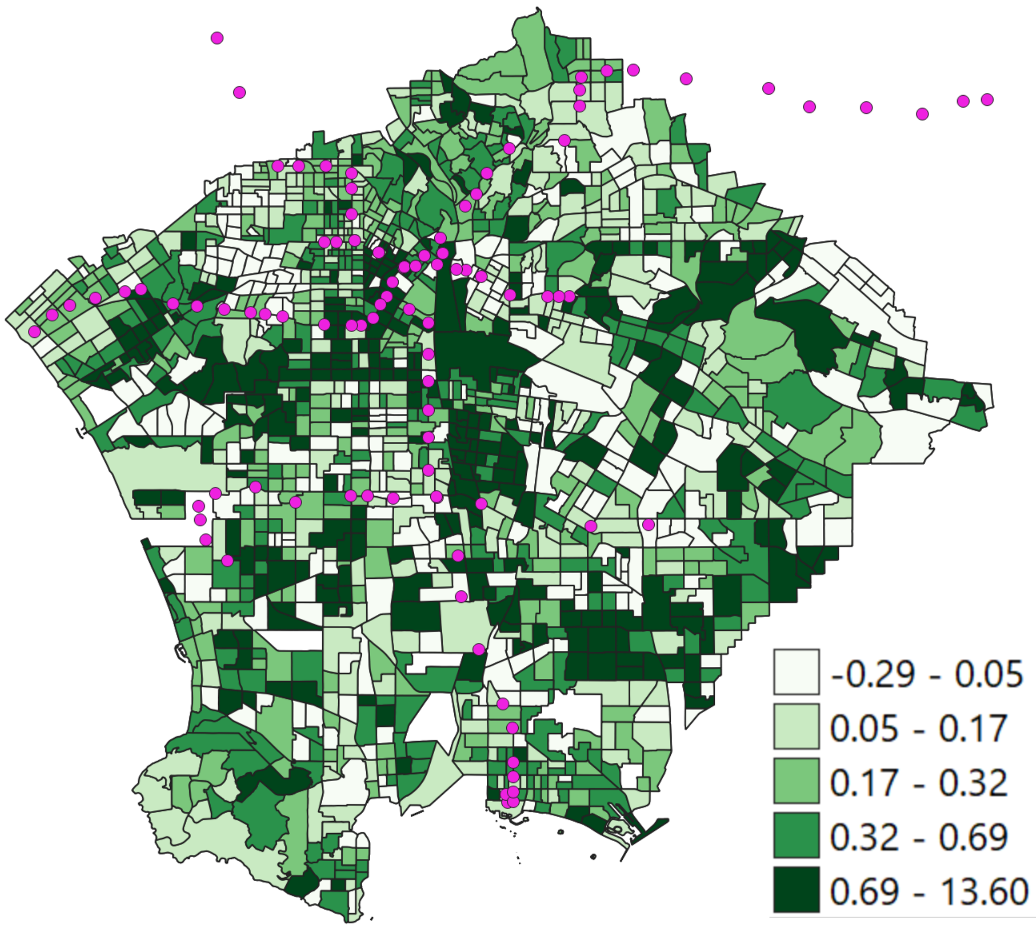


Figure 2.19: Southern Los Angeles County Difference in Public Transit Share by Census Tract

Figures (2.17) to (2.18) plot shares by census tracts (origins) for south Los Angeles county. Darker shades of blue represent tracts where public transit has higher shares. Metro LA rail stations are represented by pink dots. Figure (2.19) shows changes in shares after the implementation of the Expo line expansion. Darker shades of green show higher changes. Tracts along the Expo line, down town Los Angeles and along the Blue line (connecting down town Los Angeles to Long Beach, brown line in figure (2.14)) are the tracts that see the most increment in public transit share as a consequence of the Expo line extension.

2.7 The Value of The Rail System in Los Angeles County

In this counterfactual I use the model to find the value of the Los Angeles Metro Rail. The Los Angeles Metro rail system consists of 2 subway lines, 4 light rail lines, and 93 stations connected by 97.6 miles of rails (see figure (2.14))[13]. The rail system is Los Angeles has an average weekday ridership of 333,287 passengers in 2016[14]. On top the of rail system in the county of Los Angeles there is a bus system composed of 140 lines, 13,978 bus stations covering 1,433 road miles. The average weekday ridership of the bus system in 2016 is of 1,024,267 passengers[15]. However, I assume that the bus system keeps working with the same schedule as in the benchmark case. Therefore, commuters still have a public transit alternative to rail.

I modify the GTFS file to eliminate all rail lines and then I simulate the public transit system and recompute all public transit travel time and cost for all origin-destination pairs. Removing the rail system affects travel times and costs of 54,392 public transit trips, this represents 20.94% of all origin-desitnation trips. Then I simulate the model with these new travel times and costs and compare it to the benchmark model. Table (2.8) shows the results of this counterfactual.

Table 2.8: No Rail System Counterfactual Results

	Benchmark (1)	No Rail (2)	Difference (3)
Car Share (%)	77.95	79.83	1.88
Bus Share (%)	17.23	15.32	-1.91
Walk Share (%)	4.82	4.85	0.03
Total Time (hours)	597,698.54	609,070.58	11,372.04
Welfare	-1,786,697.15	-1,794,721.34	-8,024.19

[13] Los Angeles Metro Facts at a Glance
[14] Metro LAestimated ridership stats
[15] Metro LAestimated ridership stats

When I eliminate the rail network from the public transit system, the public transit share decreases by 1.91 percentage points. Almost all commuters change (1.88 percentage points) divert towards car mode and walking share remains practically unchanged (0.03 percentage points). The total system travel time, the aggregate time needed to complete the city trips increases by 11,372 hours (1.9% increase in travel time). Finally, the welfare change due to the removal of the rail network in Los Angeles county is -0.45%.

To annual value of the rail service in the Los Angeles county I make the same assumptions as in the previous counterfactual:

$$V_{rail} = 16 * 2 * 260 * 11372 * 19.81 = 1,874,323,942$$

I find that the value of the rail system, in terms of time saved per year is of almost $1.9 Billion. This result is in line with the findings in (Anderson, 2014 ([**?**])). He uses a regression discontinuity design to exploit a sudden strike in 2003 by Los Angeles transit workers and find that the annual value of the public transit system is between $1.2 billion to $4.1 billion. My estimate is in the lower side of his range estimate since I'm only considering the rail system. As a last exercise I remove the entire public transit system and find that its annual value is of $7.2 billion in 2016 (see results in table (2.10))[16].

[16]When congestion is not taken into account, I find the annual value of the rail system to be of $705,673,404 and the value of the entire public transit system to be of $1,093,080,934. Again, this goes in line with the findings in (Anderson, 2014 ([**?**])) where he finds large congestion relief benefits of public transit than previous research because his setting allows to account for congestion externalities. Here, I explicitly introduce the traffic congestion mechanism which allows to generate the same effect that he is able to estimate by means of a reduced form approach. This makes me be confident on my results.

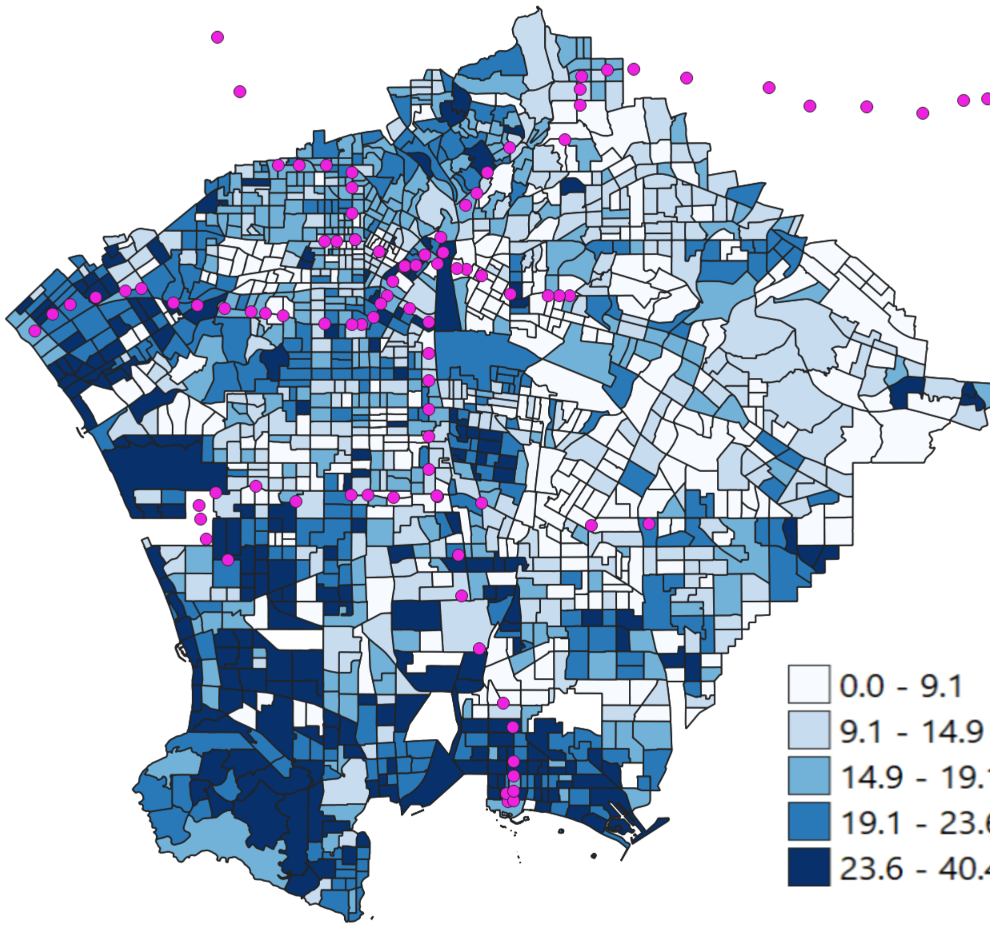


Figure 2.20: Southern Los Angeles County Public Transit Share by Census Tract without Light Rail System

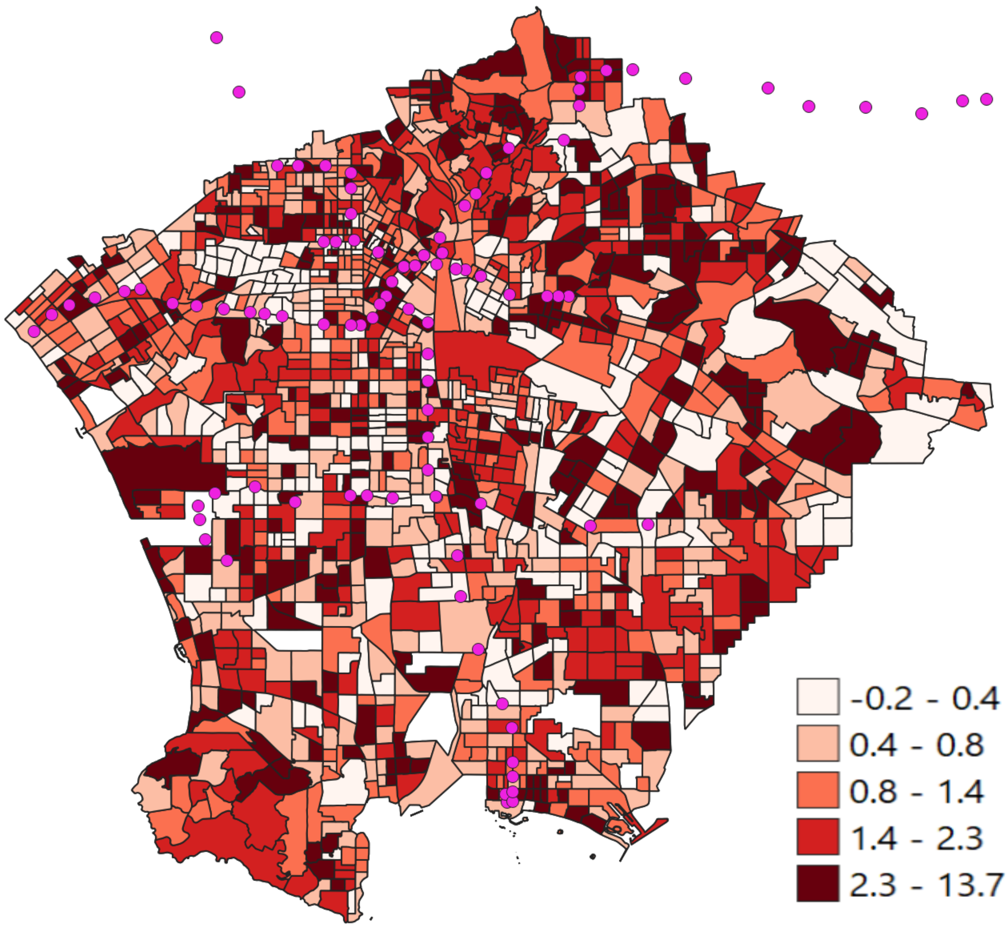


Figure 2.21: Southern Los Angeles County Difference in Public Transit Share by Census Tract

2.8 Conclusion

In this paper I bring the structural model of the commuting market developed in Marin-Aranega a (1) to the data. I start by estimating demand and supply parameters. Then I evaluate the accuracy of the model against observed data and find that the model is accurate. Finally I use the estimated model to evaluate two different counterfactuals: first, the effect of the Expo line extension to Santa Monica in Los Angeles county, and second, the value of the rail system in Los Angeles county.

To estimate demand parameters I use a mix of the 2012 California Household Travel

Survey and Google data. With demand parameters I find that the estimated value of time is around $20 per hour which is in line with the median hourly wage. On the supply side I estimate congestion parameters for the Los Angeles county highway system. I find that congestion kicks in for lower car volumes than for standard parameter estimates used in the literature.

With model parameter estimates, the last step before applying the model is to asses it's accuracy. I use road network data from the SCAG, METRO LA's transit feed for the public transit system, and travel demand data from LODES and the ACS to simulate a typical 8am weekday commute in Los Angeles county. Then I compare the results from the model to the data and find that the model is able to capture mean commuting travel times, mean travel times by commuting mode, and mode shares accurately.

The first counterfactual evaluates the effect and welfare impacts of the Expo rail line extension that connects Santa Monica to Los Angeles. I find that this public infrastructure investment, in the presence of traffic congestion, increases public transit share by 0.68 points, affects almots 89% of all commutes in Los Angeles county, saves 1427 commuting hours every 30 minutes, and increases welfare by 0.085%. With this numbers, the county needs 6.37 years to recover its investment of $1.51 billion. However, as expected, when traffic congestion is not taken into account, the effect of the investment is higher, the welfare effect is 2.3 times higher and the time to recover investment is 2.67 years. This brings light to the fact that, when evaluating this kind of investments, taking into account the effect of traffic congestion is important, and can change policy choices.

In the second counterfactual I estimate the value of the rail system in Los Angeles county. I find that the rail system has an annual value of $1.9 billion in 2016. Moreover, in the appendix I extend this counterfactual to estimate the effect of the entire public transit system. I find its annual value to be of $7.2 billion in 2016.

2.9 Appendix

2.9.1 Appendix 1: The effect of Expo line extension by distance to rail station

In this section I investigate the heterogeneity of the effects of the Expo line extension by distance to a rail station. I define 4 types of census tracts: all within 1/4 mile to a station, all within 1/2, all within 1 mile and all tracts. See figures (2.22) to (2.24) for a graphical representation of these groups.

The next table shows the results of repeating the analysis of the main text but just using the census tracts belonging to the different groups. The main effect is within half a mile of a rail station. This goes in line to the half-mile definition of catchment are for public transit[17].

The following maps show the census tracts used in each of the analysis (in yellow). Rail stations are depicted as pink dots.

[17] From Guerra, Cervero and Tischler (2012) ([**?**]): "*The 0.5-mile distance has become accepted for gauging a transit station's catchment area in the United States and is the de facto standard for the planning of U.S. transit-oriented developments*"

Table 2.9: Effect of Expo Line Extension by Distance to Rail Stations

	Census Tract Group			
	1/4 mile	1/2 mile	1 mile	All
	(1)	(2)	(3)	(4)
Bus Share Diff.	0.73	0.76	0.73	0.68

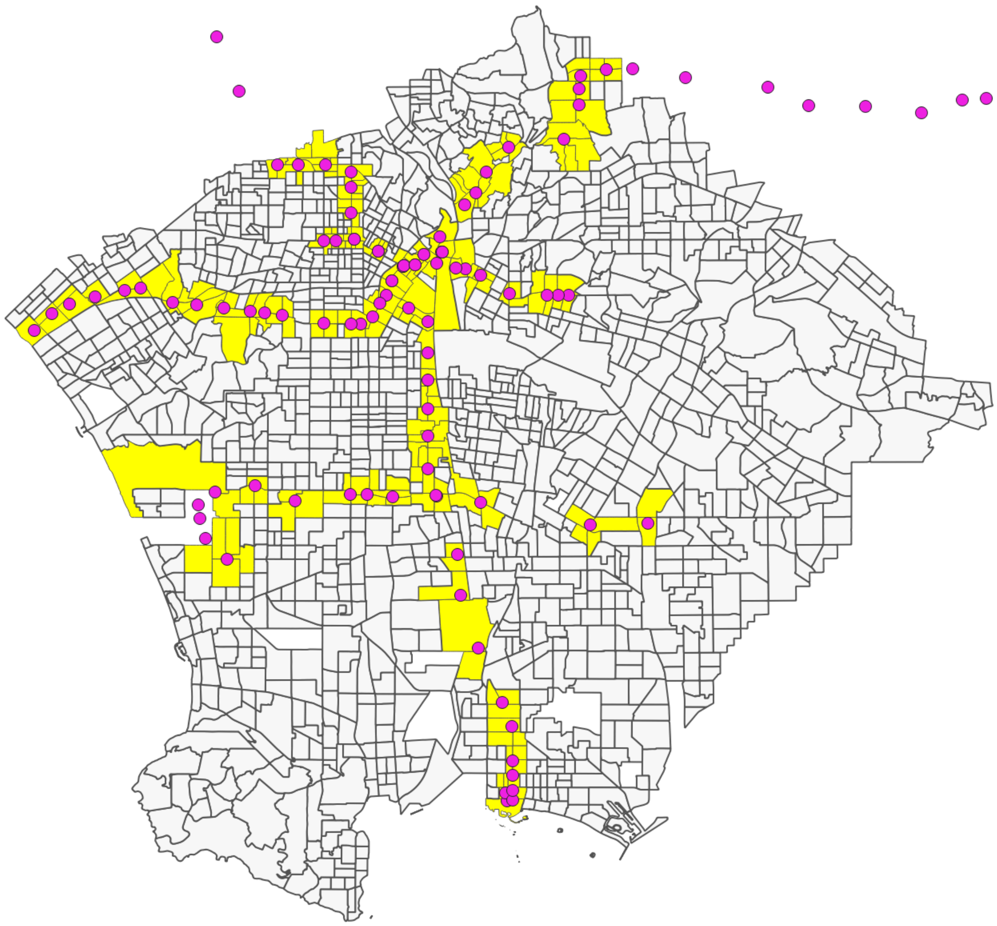


Figure 2.22: Tracts In 1/4 Mile of Rail Stations

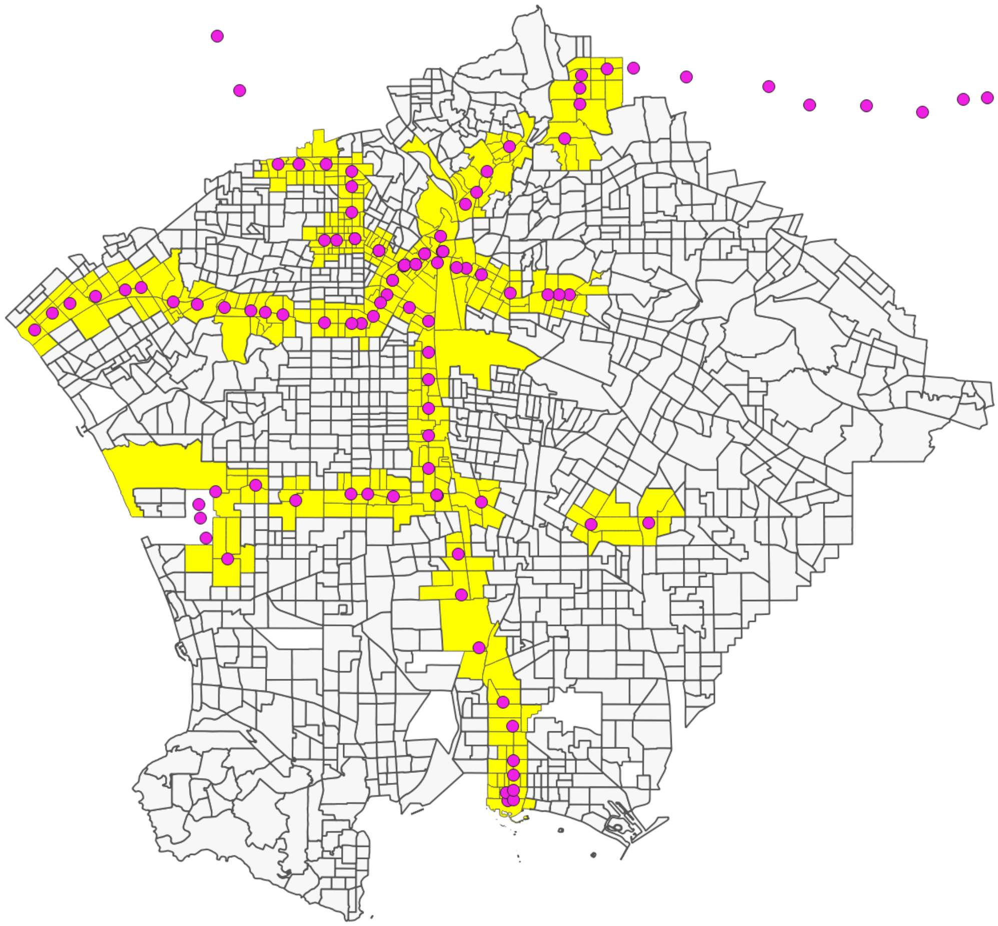


Figure 2.23: Tracts In 1/2 Mile of Rail Stations

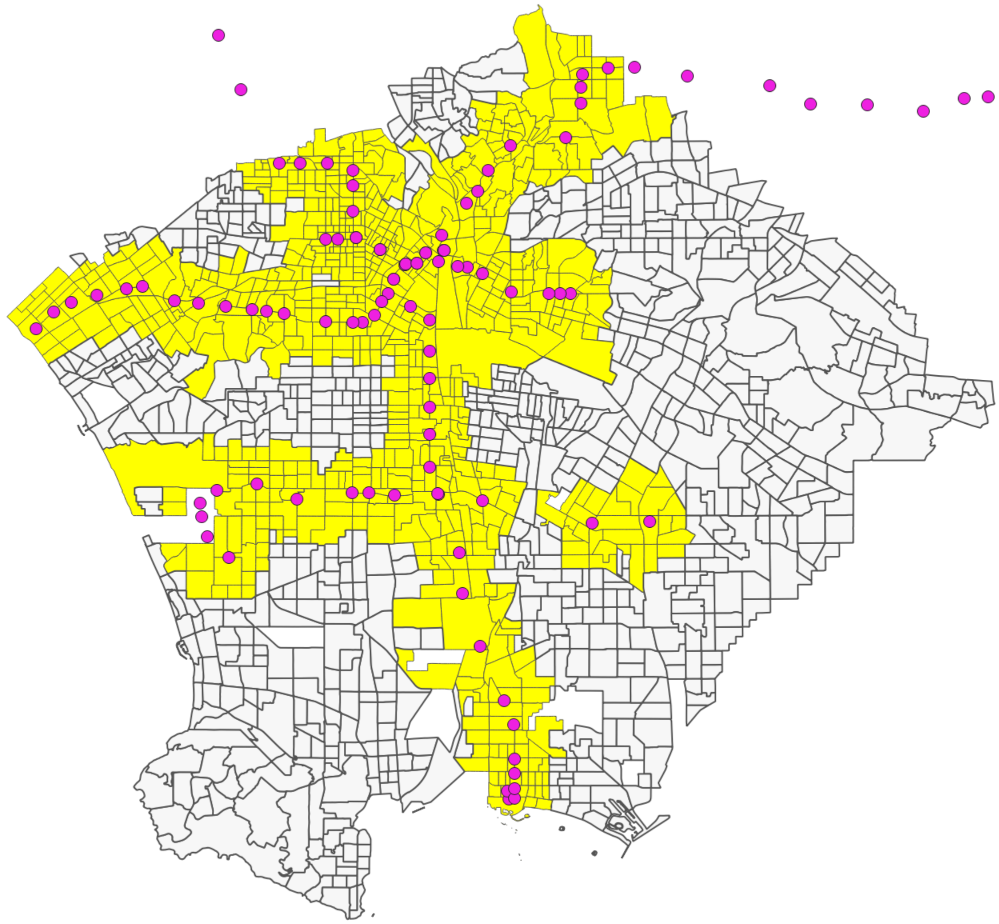


Figure 2.24: Tracts In 1 Mile of Rail Stations

2.9.2 Appendix 2: The value of the public transit system in Los Angeles county

In this section I repeat the analysis of the second counterfactual in the main text but in this case I eliminate the entire transit system from the cummert's choice set. Public transit share is zero and car share and walk share increase by 10.5% and 6.7% respectively. Total system travel time increases by approximately 89,000 hours and welfare decreases by 9.75%. Using the same calculations as in the main text I find that the yearly value of the transit system is of $7.2 billion.

One thing to consider is that the bus system represents the vast majority of the public transit system in terms of stations and miles covered. Hence, when I eliminate the rail system, commuters using public transit still have plenty of bus alternatives to choose from. This is not the case when eliminating the entire system. Another interesting counterfactual would be to remove the bus system and leave the rail system. However, this is left as a future exercise.

Table 2.10: No Public Transit System Counterfactual Results

	Benchmark (1)	No Transit (2)	Difference (3)
Car Share (%)	77.95	88.42	10.47
Bus Share (%)	17.23	0	-17.23
Walk Share (%)	4.82	11.58	6.76
Total Time (hours)	597,698.54	686,578.21	88,879.67
Welfare	-1,786,697.15	-1,961,048.12	-174,350.97

2.9.3 Appendix 3: Other maps and figures

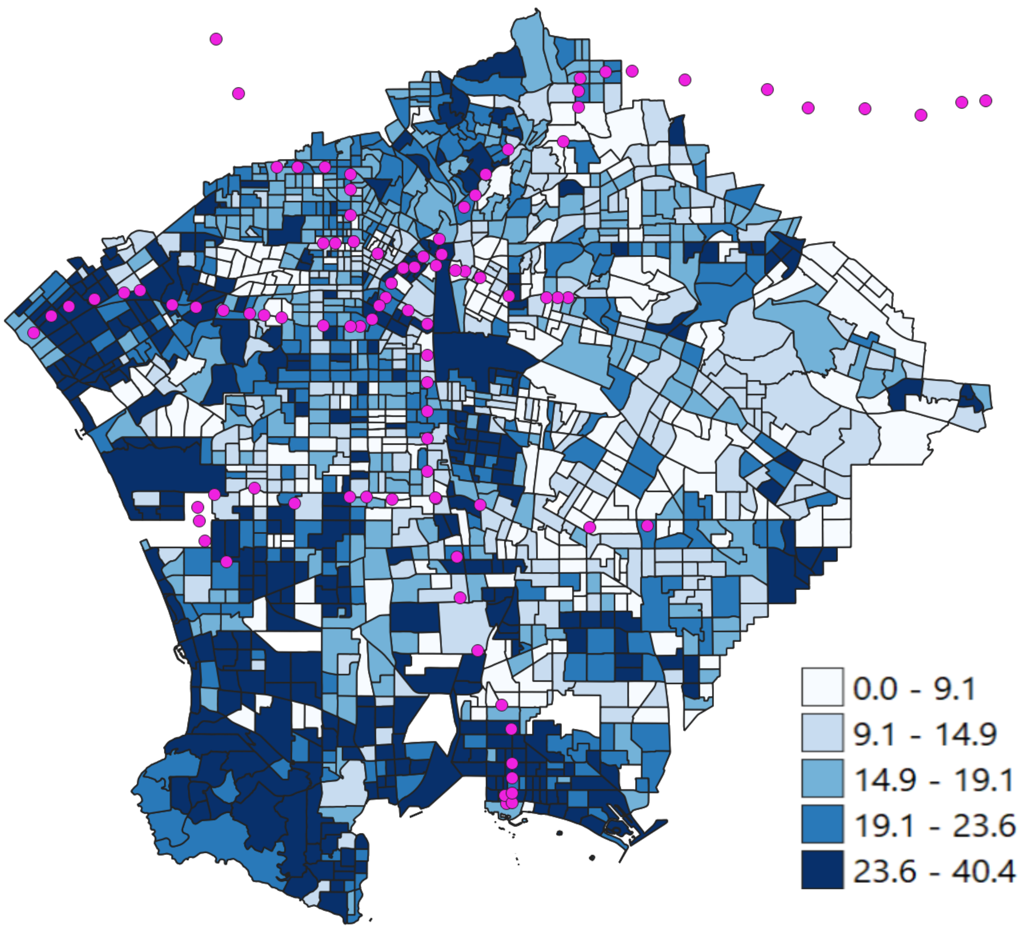


Figure 2.25: Southern Los Angeles County Public Transit Share by Census Tract with No Congestion

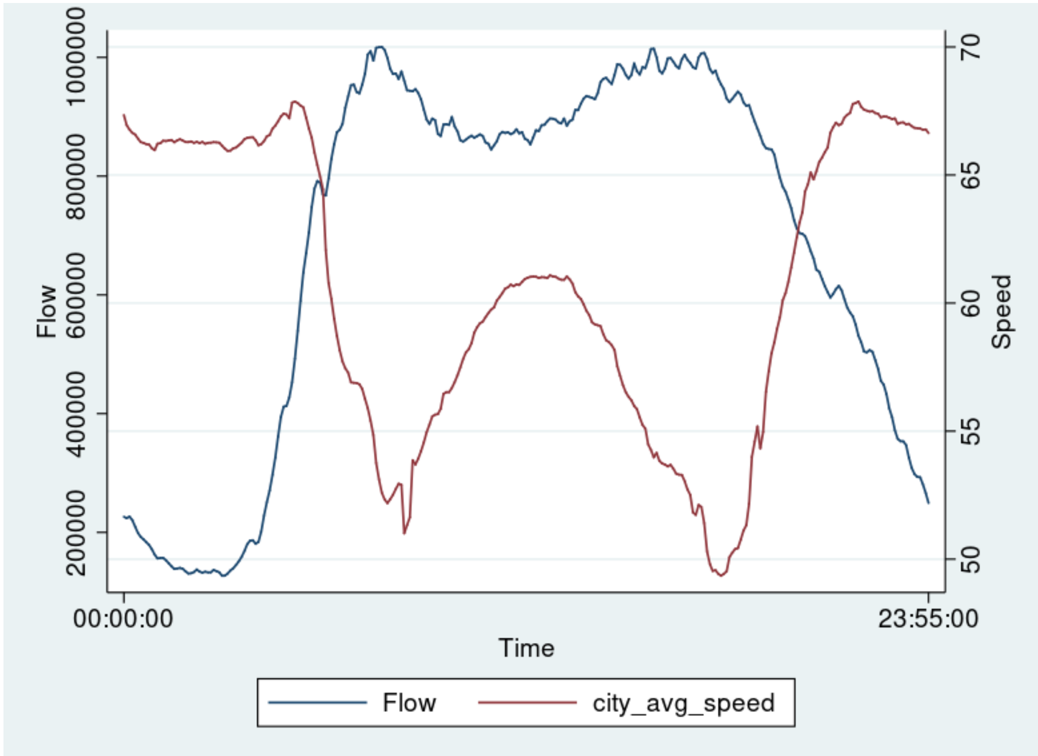


Figure 2.26: Los Angeles County Flow and Speed Highway System: Weekday

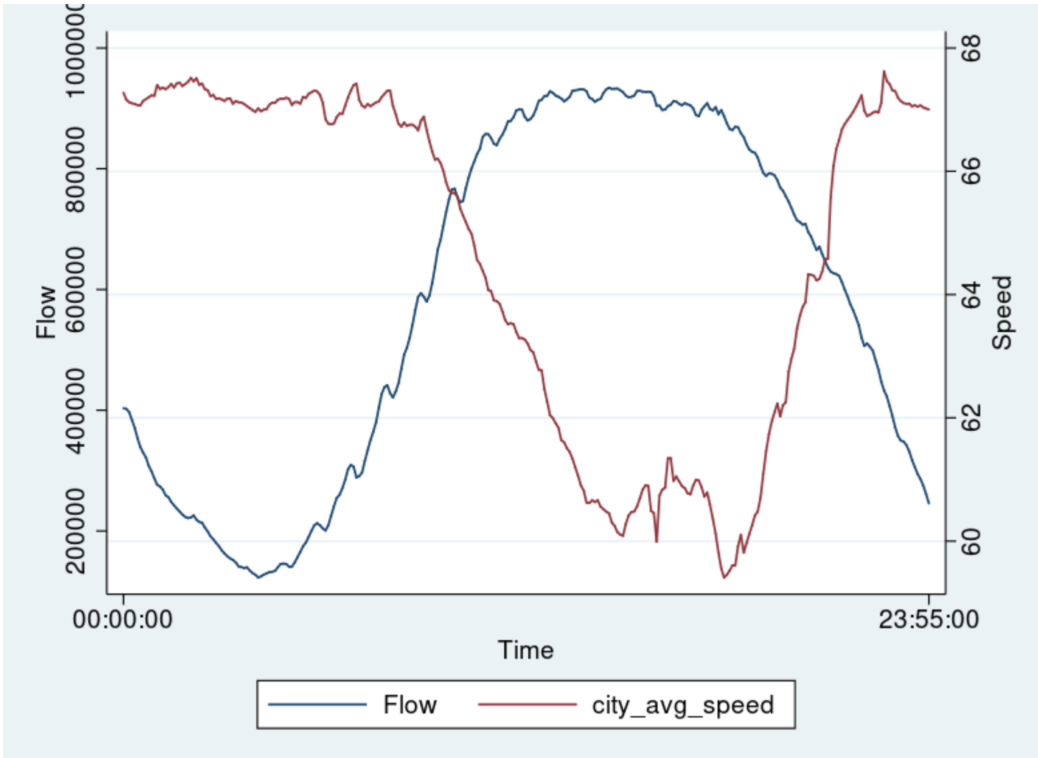


Figure 2.27: Los Angeles County Flow and Speed Highway System: Weekend

2.9.4 Appendix 4: Congestion function parameters: estimated v. proposed by the BPR. Results table

	Benchmark		Highway		Difference		
Link	Flow	Travel Time	Flow	Travel Time	Flow	Time	Percent
1	4362.078819	6.000724	4800.137673	6.001062	-438.058854	-0.000338	1.000056
2	7852.385574	4.007604	8552.558948	4.010701	-700.173374	-0.003097	1.000773
3	4711.500869	6.000986	5153.012325	6.001410	-441.511456	-0.000425	1.000071
4	5686.965779	6.298058	5826.147524	6.429874	-139.181744	-0.131815	1.020930
5	7511.068409	4.006366	8201.310527	4.009048	-690.242118	-0.002683	1.000670
6	14387.968086	4.299982	15637.631469	4.418583	-1249.663383	-0.118601	1.027582
7	11346.015646	4.033144	12187.398502	4.044124	-841.382855	-0.010980	1.002722
8	14246.415413	4.288350	15699.214195	4.425216	-1452.798782	-0.136866	1.031916
9	18437.557031	2.346685	19055.214484	2.395527	-617.657453	-0.048843	1.020813
10	6138.559305	8.200886	6339.845470	8.504070	-201.286166	-0.303184	1.036970

11	18333.768795	2.338944	18468.043625	2.348983	-134.274831	-0.010039	1.004292
12	9659.098960	12.713257	8755.173503	9.881568	903.925458	2.831690	0.777265
13	14818.920685	8.616826	16289.259694	10.280398	-1470.339008	-1.663572	1.193061
14	6052.597599	6.665478	6182.274637	6.812863	-129.677038	-0.147385	1.022112
15	9388.697924	11.777778	8570.535683	9.400898	818.162241	2.376880	0.798189
16	13096.820930	17.328557	12604.704093	15.151290	492.116836	2.177268	0.874354
17	12913.442473	6.309140	13355.322145	6.785859	-441.879672	-0.476719	1.075560
18	14258.816467	2.041337	15541.128253	2.058335	-1282.311785	-0.016999	1.008327
19	13167.737058	17.663265	12771.314694	15.860540	396.422363	1.802725	0.897939
20	12271.204871	5.698335	12887.533196	6.282662	-616.328324	-0.584327	1.102544
21	6367.955457	13.791922	6790.810501	14.903947	-422.855043	-1.112025	1.080629
22	8435.697607	10.858917	8493.262232	11.020485	-57.564625	-0.161568	1.014879
23	14945.009059	8.741502	15878.595500	9.767708	-933.586441	-1.026205	1.117395
24	6771.577515	14.848626	7425.446985	17.010512	-653.869470	-2.161885	1.145595
25	22286.145386	5.960200	22049.882160	5.836654	236.263226	0.123546	0.979271
26	22899.646047	6.299866	22368.975758	6.004455	530.670289	0.295412	0.953108
27	18345.049926	13.494481	18837.818620	14.444603	-492.768694	-0.950122	1.070408
28	23267.111287	13.912863	22626.356131	13.076561	640.755156	0.836302	0.939890
29	10952.845470	19.542884	11323.621360	21.756817	-370.775890	-2.213933	1.113286
30	8285.209494	17.094342	8168.253941	16.591604	116.955553	0.502738	0.970590
31	6703.297752	9.129578	7082.094131	9.899227	-378.796379	-0.769649	1.084303
32	17486.772711	12.012937	17969.869008	12.820615	-483.096298	-0.807678	1.067234
33	8648.672244	14.672203	8225.548157	13.095632	423.124087	1.576571	0.892547
34	9746.469379	13.574248	10375.256693	16.294498	-628.787314	-2.720250	1.200398
35	11689.278464	4.037341	12044.521686	4.042091	-355.243222	-0.004750	1.001177
36	8745.262347	15.066153	8561.697103	14.328583	183.565244	0.737570	0.951045
37	14256.924336	3.041315	14501.172874	3.044219	-244.248538	-0.002905	1.000955
38	14253.749946	3.041278	14589.539909	3.045307	-335.789963	-0.004029	1.001325
39	10902.892936	16.618811	10964.745621	16.907606	-61.852685	-0.288795	1.017378
40	8921.418085	10.721264	9818.259088	13.859465	-896.841003	-3.138201	1.292708
41	8682.793926	11.166911	8646.291584	11.063861	36.502341	0.103050	0.990772
42	7480.925680	7.194637	8948.206580	10.539494	-1467.280900	-3.344857	1.464910
43	24225.387324	15.299223	23203.309513	13.826426	1022.077812	1.472797	0.903734
44	7736.248808	8.886435	8916.176342	11.857161	-1179.927534	-2.970725	1.334299
45	18998.166211	4.302707	18433.822804	4.154680	564.343407	0.148027	0.965597
46	18785.718599	9.600656	17666.983447	8.163278	1118.735152	1.437379	0.850283

47	8652.172202	11.483865	8147.769220	10.099038	504.402982	1.384827	0.879411
48	11098.865659	20.388460	11565.191597	23.321244	-466.325938	-2.932784	1.143845
49	11776.293195	9.712209	11321.949790	8.589148	454.343405	1.123061	0.884366
50	16175.403851	3.205372	19133.176652	3.402040	-2957.772801	-0.196668	1.061356
51	8552.691144	18.326864	8436.773532	17.778289	115.917612	0.548575	0.970067
52	11734.081158	9.602225	11261.242664	8.448959	472.838494	1.153266	0.879896
53	9713.662539	6.932213	9792.032828	7.093323	-78.370289	-0.161110	1.023241
54	15962.794031	2.064929	16075.592669	2.066783	-112.798638	-0.001855	1.000898
55	15477.846284	3.172172	18759.836128	3.371567	-3281.989844	-0.199395	1.062858
56	19654.040883	4.298427	20140.512983	4.329088	-486.472100	-0.030662	1.007133
57	18942.496592	4.287505	18131.062372	4.080670	811.434220	0.206835	0.951759
58	9938.932152	7.405908	9999.845293	7.539657	-60.913141	-0.133749	1.018060
59	9109.137607	10.595908	8997.321548	10.277959	111.816059	0.317949	0.969993
60	19498.720918	4.289104	20024.377706	4.321563	-525.656789	-0.032459	1.007568
61	9278.737601	11.101026	8902.373582	10.017121	376.364019	1.083904	0.902360
62	6654.662541	8.692618	6481.612197	8.423275	173.050344	0.269343	0.969015
63	7863.497984	9.320575	8056.165422	9.759836	-192.667438	-0.439260	1.047128
64	6610.401396	8.621693	6597.532370	8.601337	12.869026	0.020356	0.997639
65	9695.017504	5.542736	8753.884375	4.354765	941.133129	1.187971	0.785671
66	10083.121323	11.165957	9595.740669	9.697938	487.380654	1.468019	0.868527
67	18753.119137	9.554958	18716.582018	9.504023	36.537119	0.050936	0.994669
68	7822.039156	9.230176	7629.162005	8.828122	192.877151	0.402053	0.956441
69	9464.531057	5.217665	8896.798414	4.512346	567.732644	0.705319	0.864821
70	9353.394082	11.347645	9455.373585	11.673368	-101.979502	-0.325723	1.028704
71	7602.419504	7.407277	8121.324217	8.437185	-518.904713	-1.029908	1.139040
72	9048.849347	10.436424	10220.882779	14.476709	-1172.033431	-4.040284	1.387133
73	7589.419697	3.496273	8662.532398	4.539546	-1073.112701	-1.043273	1.298396
74	10799.718547	16.147900	10953.112656	16.852916	-153.394110	-0.705016	1.043660
75	10269.346624	11.786145	9568.746803	9.622887	700.599821	2.163258	0.816458
76	7406.368786	3.357056	8601.159229	4.468337	-1194.790443	-1.111281	1.331028

Table 2.11: BPR v. Estimated Parameters in the Sioux Falls Equilibrium

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
1	0.0	100.0	100.0	500.0	200.0	300.0	500.0	800.0	500.0	1300.0	500.0	200.0	500.0	300.0	500.0	500.0	400.0	100.0	300.0	300.0	100.0	400.0	300.0	100.0
2	100.0	0.0	100.0	200.0	100.0	400.0	200.0	400.0	200.0	600.0	200.0	100.0	300.0	100.0	100.0	400.0	200.0	0.0	100.0	100.0	0.0	100.0	0.0	0.0
3	100.0	100.0	0.0	200.0	100.0	300.0	100.0	200.0	100.0	300.0	300.0	200.0	100.0	100.0	100.0	200.0	100.0	0.0	0.0	0.0	0.0	100.0	100.0	0.0
4	500.0	200.0	200.0	0.0	500.0	400.0	400.0	700.0	700.0	1200.0	1400.0	600.0	600.0	500.0	500.0	800.0	500.0	100.0	200.0	300.0	200.0	400.0	500.0	200.0
5	200.0	100.0	100.0	500.0	0.0	200.0	200.0	500.0	800.0	1000.0	500.0	200.0	200.0	100.0	200.0	500.0	200.0	0.0	100.0	100.0	100.0	200.0	100.0	0.0
6	300.0	400.0	300.0	400.0	200.0	0.0	400.0	800.0	400.0	800.0	400.0	200.0	200.0	100.0	200.0	900.0	500.0	100.0	200.0	300.0	100.0	200.0	100.0	100.0
7	500.0	200.0	100.0	400.0	200.0	400.0	0.0	1000.0	600.0	1900.0	500.0	700.0	400.0	200.0	500.0	1400.0	1000.0	200.0	400.0	500.0	200.0	500.0	200.0	100.0
8	800.0	400.0	200.0	700.0	500.0	800.0	1000.0	0.0	800.0	1600.0	800.0	600.0	600.0	400.0	600.0	2200.0	1400.0	300.0	700.0	900.0	400.0	500.0	300.0	200.0
9	500.0	200.0	100.0	700.0	800.0	400.0	600.0	800.0	0.0	2800.0	1400.0	600.0	600.0	600.0	900.0	1400.0	900.0	200.0	400.0	600.0	300.0	700.0	500.0	200.0
10	1300.0	600.0	300.0	1200.0	1000.0	800.0	1900.0	1600.0	2800.0	0.0	4000.0	2000.0	1900.0	2100.0	4000.0	4400.0	3900.0	700.0	1800.0	2500.0	1200.0	2600.0	1800.0	800.0
11	500.0	200.0	300.0	1500.0	500.0	400.0	500.0	800.0	1400.0	3900.0	0.0	1400.0	1000.0	1600.0	1400.0	1400.0	1000.0	100.0	400.0	600.0	400.0	1100.0	1300.0	600.0
12	200.0	100.0	200.0	600.0	200.0	200.0	700.0	600.0	600.0	2000.0	1400.0	0.0	1300.0	700.0	700.0	700.0	600.0	200.0	300.0	400.0	300.0	700.0	700.0	500.0
13	500.0	300.0	100.0	600.0	200.0	200.0	400.0	600.0	600.0	1900.0	1000.0	1300.0	0.0	600.0	700.0	600.0	500.0	100.0	300.0	600.0	600.0	1300.0	800.0	800.0
14	300.0	100.0	100.0	500.0	100.0	100.0	200.0	400.0	600.0	2100.0	1600.0	700.0	600.0	0.0	1300.0	700.0	700.0	100.0	300.0	500.0	400.0	1200.0	1100.0	400.0
15	500.0	100.0	100.0	500.0	200.0	200.0	500.0	600.0	1000.0	4000.0	1400.0	700.0	700.0	1300.0	0.0	1200.0	1500.0	200.0	800.0	1100.0	800.0	2600.0	1000.0	400.0
16	500.0	400.0	200.0	800.0	500.0	900.0	1400.0	2200.0	1400.0	4400.0	1400.0	700.0	600.0	700.0	1200.0	0.0	2800.0	500.0	1300.0	1600.0	600.0	1200.0	500.0	300.0
17	400.0	200.0	100.0	500.0	200.0	500.0	1000.0	1400.0	900.0	3900.0	1000.0	600.0	500.0	700.0	1500.0	2800.0	0.0	600.0	1700.0	1700.0	600.0	1700.0	600.0	300.0
18	100.0	0.0	0.0	100.0	0.0	100.0	200.0	300.0	200.0	700.0	200.0	200.0	100.0	100.0	200.0	500.0	600.0	0.0	300.0	400.0	100.0	300.0	100.0	0.0
19	300.0	100.0	0.0	200.0	100.0	200.0	400.0	700.0	400.0	1800.0	400.0	300.0	300.0	300.0	800.0	1300.0	1700.0	300.0	0.0	1200.0	400.0	1200.0	300.0	100.0
20	300.0	100.0	0.0	300.0	100.0	300.0	500.0	900.0	600.0	2500.0	600.0	500.0	600.0	500.0	1100.0	1600.0	1700.0	400.0	1200.0	0.0	1200.0	2400.0	700.0	400.0
21	100.0	0.0	0.0	200.0	100.0	100.0	200.0	400.0	300.0	1200.0	400.0	300.0	600.0	400.0	800.0	600.0	600.0	100.0	400.0	1200.0	0.0	1800.0	700.0	500.0
22	400.0	100.0	100.0	400.0	200.0	200.0	500.0	500.0	700.0	2600.0	1100.0	700.0	1300.0	1200.0	2600.0	1200.0	1700.0	300.0	1200.0	2400.0	1800.0	0.0	2100.0	1100.0
23	300.0	0.0	100.0	500.0	100.0	100.0	200.0	300.0	500.0	1800.0	1300.0	700.0	800.0	1100.0	1000.0	500.0	600.0	100.0	300.0	700.0	700.0	2100.0	0.0	700.0
24	100.0	0.0	0.0	200.0	0.0	100.0	100.0	200.0	200.0	800.0	600.0	500.0	700.0	400.0	400.0	300.0	300.0	0.0	100.0	400.0	500.0	1100.0	700.0	0.0

Table 2.12: Origin-Destination Demand Matrix For Sioux Falls, SD

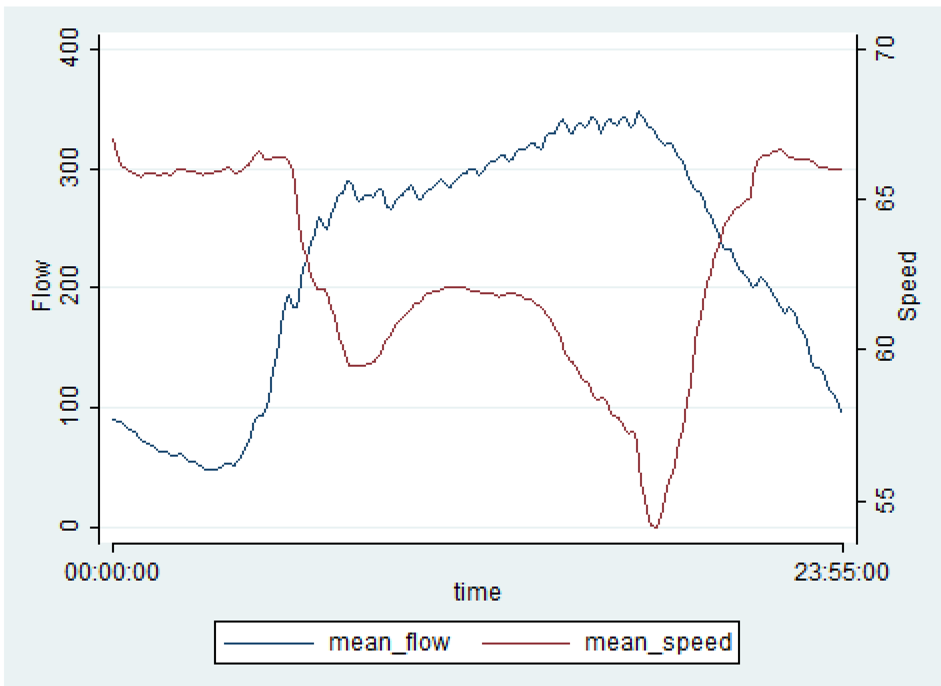


Figure 2.28: Congestion in Los Angeles

Chapter 3

Population Density, Traffic Congestion, and Commuting Mode Choice: Theory and Empirical Evidence

3.1 Introduction

In this paper I explore the relationship between population density and commuting mode choice motivated by the public debate on the elimination of single-family zoning. First, I show evidence that geographical areas where population density is higher tend to use less the car as a commuting alternative. Next, I show that commuters substitute almost 1-to-1 car by public transit alternatives and vice versa. Then, I extend the model of internal city structure by Ahlfeldt et. al. (2015) ([?]) to allow, in a second stage, for mode choice and routing that generates traffic congestion and endogenous travel times. Finally, I run simulations of the second stage of the model for different population densities in the county of Los Angeles. The model captures both facts: the higher density the less use of the car, and this translates almost 1-to-1 into increases in public transit usage.

With the increase of population and the concentration of economic activity in urban areas, local governments are passing and considering laws ending with single-family zoning and encouraging more dense development. Minneapolis, where 70% of residential land is under singe-family zoning laws, passed a citywide upzoning law in 2018. Cities like Portland (77%), Seattle (81%), and San Jose (94%) have passed similar laws. See figure (3.11) for examples of US cities with different single-family zoning proportions over total residential land.

The increase in housing supply will lower prices and attract more population to the city. This effect will spillover to the surrounding areas (Tanure Veloso, 2020 [?]) generating a denser metropolitan area. However, to the best of my knowledge, no paper has studied the effect of this increase in density on the metropolitan commuting market. More people implies more travel and more traffic congestion. Traffic congestion increases car travel times and costs making commuting by car comparatively less attractive to commuting by public transit or walking (see the first two chapters of this dissertation 1, 2).

First I document the relation between population density and car mode choice. I find that a 20% increase in population density (approximately 1,058 persons per square mile) translates in a decrease between 0.7 and 0.95 percentage points in car mode share. Next, I show that changes in car share translate into inverse changes in public transit use, that is, commuters don't consider other commuting modes such as walking.

To translate changes in population density to changes in mode shares I extend Ahlfeldt et. al. (2015) ([?]) model of internal city structure to allow for mode choice and endogenous travel times. Their model features agglomeration forces, in terms of residential and employment amenities, and dispersion forces, land prices. Consumers want to locate in the same, more attractive, locations but as they do land prices

increase making them to disperse.
Travel times are an agglomeration force, since all else equal, consumers prefer to have their employment and residential locations close to each other to pay a lower cost. Their model can not generate the link between density and mode choice because travel times are fixed. Even if there was a mode choice, shares would remain unchanged as there are no changes in travel times.

To overcome this limitation I follow (the model developed in the first chapter 1) and introduce a second stage to their model where, given a travel demand matrix, consumers have a commuting mode choice and a routing choice. Moreover, on the supply side, there is a road transportation network that gets congested as a function of network use, that is, as more consumers want to use a given road segment travel times in that segment increase. Hence, generating endogenous travel times.

Finally, I use the second stage of the model to investigate how different levels of uniform increase in population density affect mode choice and travel times in Los Angeles county. I chose Los Angeles county for two main reasons: first, it is one of the cities with the worst traffic in the US and, second, the city of Los Angeles is considering a change in single-family zoning which affects more than 73% of all residential land (see figure (3.1) and (3.12)).
Using data and parameter estimates from the second chapter (2) I run simulations of the model for different levels of travel demand which I take from the data. The second stage model is able to reproduce the observed changes in the data: a 20% population increase translates into a 0.82 percentage points decrease in car commuting share. Moreover, this decrease is absorbed 90.3% by an increase in public transit use.

Figure 3.1: Single Family zoning in Los Angeles city.

Related Literature

This paper relates to two strands of the literature: on the one hand, with the quantitative spatial economics and on the other hand, the economics of traffic congestion.

First of all the paper is related to the recent literature on quantitative spatial models. Models like Allen and Arkolakis (2014) ([?]), Ahlfeldt et. al. (2015) ([?]), Allen, Arkolakis and Li (2017) ([?]) or Monte, Redding and Rossi-Hansberg (2018) ([?]). For a survey in quantitative spatial models see Redding and Rossi-Hansberg (2017) ([?]). All these models include agglomeration forces (productivity and residence spillovers) and dispersion forces (land prices). Unlike these papers, in my model travel times arise endogenously as a function of road network usage. This allows to perform counterfactuals in which general equilibrium effects are not only present through the reallocation of economic activity but also through commuting costs since changing the distribution of economic activity will change commuting travel times.

The literature of housing land regulation was surveyed in [?] and they study how housing supply affect home prices, wealth and the spatial distribution of people across locations. [?] study the origins of land-use regulation and finds that excessive local regulation reduces aggregate productivity, but not necessarily welfare because homeowners benefit from this regulation. Finally, Tanure-Veloso (2021) ([?]) studies the zoning change in the city of Minneapolis who pioneered the single family zoning deregulation. He finds that housing and renting becomes more affordable in Minneapolis and other areas of the Twin Cities as a consequence of spatial spillovers and that population density is increased. However, none of these papers study the effect of changes in zoning regulations on the commuting market.
This paper also relates the literature in traffic congestion economics pioneered by [?], and [?]. More recent papers use reduced form approached to estimate the cost of traffic congestion such [?], [?], [?], or [?].

Finally, there is a strand of the literature exploring the relationship between urban structure and commuting behavior and characteristics.[?] investigate the relationship between urban structure and commuting characteristics for US metropolitan statistical areas from 2000 to 2010. Their main findings are: (1) MSAs become more compact in terms of employment distribution, (2) more decentralized high-density areas lead to less total commuting times, while, more decentralized moderate job density areas contribute to longer commuting times, and (3) the decentralization of high job density locations is associated with less commuting time of private cars, while they have insignificant effect on commuting time of public transit. [?] study the relationship between urban density and commuting behaviour using survey data from China. They find that increases in urbanization rates is related to increase in commuting time and and increase in the public transit system for commuting.

The rest of the chapter is organized as follows: section 2 explores the relationship between population density and commuting mode choice. Section 3 lays out the theoretical model of internal city structure with endogenous commuting choice and travel times. Section 4 shows the simulation exercise and section 5 concludes.

3.2 Population Density and Commuting Mode Choice: Reduced Form Evidence

In this section I explore the relationship between commuting mode and population density in the US. For this exercise I use data coming from the 2019 American Community Survey (ACS) 5 years estimates. At different geographical levels (Counties and US Census tracts), I observe the number of commuters and number of commuters by mode. Note that I only observe work commutes since is the only statistic reported in the ACS. Therefore, all nonworking commutes are not considered in this analysis. Then I construct work commuting mode shares.

On the other hand I define population density in geography i as the ratio between population (pop_i) and land area in squares miles (sq_i):

$$density_i = \frac{pop_i}{sq_i}$$

Table (3.1) shows summary statistics of the data I am using at the County and US Census tract levels. The main difference between both geographical levels is that Census tracts are, on average, denser than Counties.

Figure (3.2) plots the relationship between population density and car commuting share. Panel (a) shows the relationship at the County level and panel (b) does the same at the Census tract level[1]. Both graphs show the same pattern: a negative

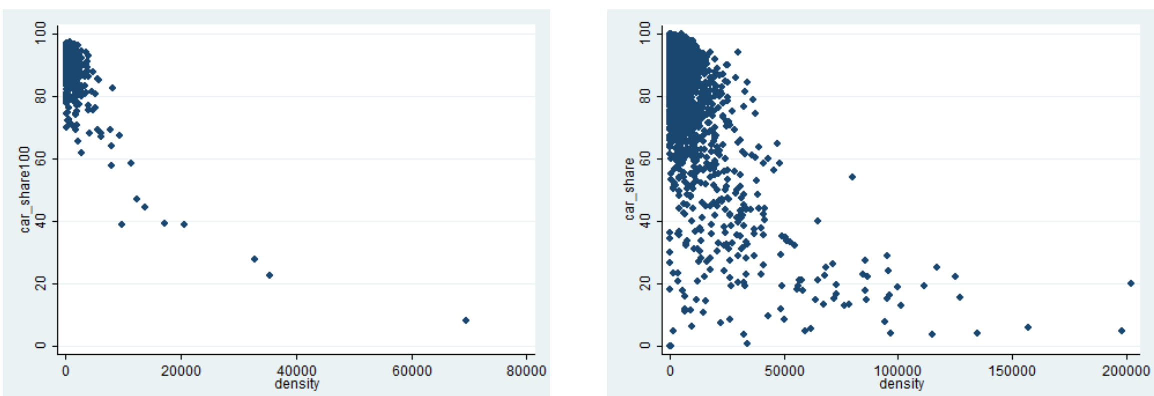
[1] I randomly select 5% of the entire sample of US Census tracts to appear in the figure. That is, 3,610 observations.

Table 3.1: Summary Statistics

	County			Census Tract		
	Car Share	Bus Share	Density	Car Share	Bus Share	Density
Mean	89.41	0.91	283.23	84.76	5.31	5292.46
Std. Dev.	7.23	3.05	1,722.66	15.59	11.65	11,855.26
Min.	7.88	0	0	0	0	0
Max.	99.08	61.24	69,468.4	100	100	510,937.5
Obs.	3,218	3,218	3,218	72,194	72,194	72,194

correlation between population density and car share.

Figure 3.2: Car Share - Density Relationship



(a) County Level

(b) Census Tract Level

Moreover, using GIS I can map US Census tracts into states and counties so that I can use fixed effects in the regression I bring to the data:

$$car_{ijs} = \beta + \alpha density_{ijs} + \rho_j + \phi_s + \epsilon_{ijs} \tag{3.1}$$

Where subindex i refers to location of consideration, and j and s to the county and state where observation i is located. Hence, ρ_j and ϕ_s are county and state fixed effects. Therefore, these fixed effects should capture things like the differences in public transit availability and service across geographies.

Table (3.2) shows the results of this analysis. In columns (1) and (2) the analysis is at the County level while in columns (3) to (5) the analysis at the US Census tract level. The estimate of the effect of population density on car share is very stable at both geographical levels even with the inclusion of fixed effects. Both are negatively correlated and the correlation is highly significant. For census tracts, an increase in population density of 20% translates into a decrease in car mode share of 0.95 percentage points using the parameter estimate of column (3) and of 0.7 using that of column (5)[2].

Table 3.2: Car Share - Density

	County		Tract		
	(1)	(2)	(3)	(4)	(5)
Density	-.167	-.164	-.09	-.072	-.066
	(-24.58)	(-31.34)	(-253.04)	(-185.43)	(-165.63)
State FX		Yes		Yes	Yes
County FX					Yes
R^2	0.158	0.545	0.470	0.573	0.593
Obs.	3,218	3,218	72,194	70,001	70,001

Coefficients per 100 people. T-statistic in parenthesis.

Next, I repeat the analysis for 6 different Metropolitan Areas. For each metropolitan area I restrict the sample to the Census tracts on that area and run a version of

[2] In the model I will simulate in section (3.4), locations are defined at the US Census tract and will be comparing the output of the simulations against those results.

equation (3.1). As above, density and car share are negatively correlated and this correlation is highly significant. However, we can see here significant variation on the parameter estimates: the correlation in the Seattle-Tacoma area is 4.67 times higher than in Los Angeles-Long Beach area.

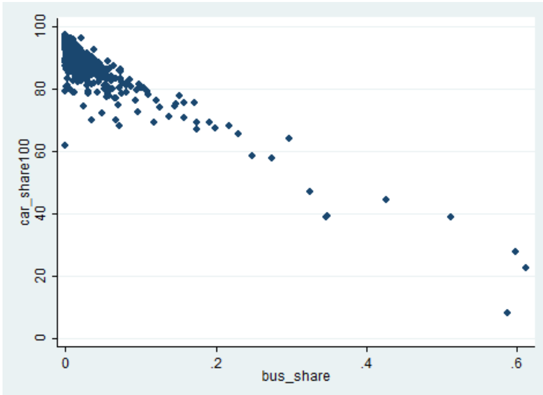
Table 3.3: Car Share - Density

	Los Angeles Long Beach	New York Newark	Chicago	Seattle Tacoma	Washington DC	Minneapolis Saint Paul
Density	-.046 (-27.08)	-.0678 (-82.90)	-.0627 (-27.72)	-.215 (-27.46)	-.146 (-33.60)	-.183 (-24.34)
R^2	0.203	0.596	0.254	0.508	0.443	0.419
Obs.	2,886	4,708	2,254	731	1,418	819

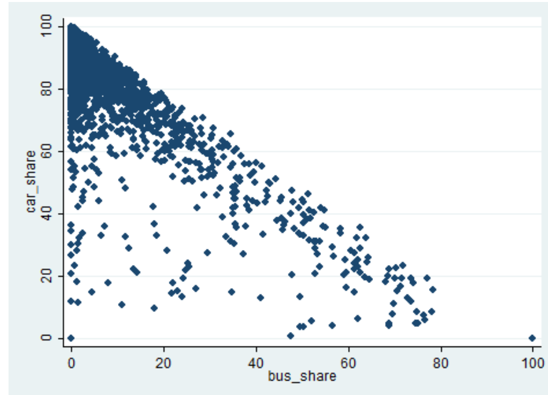
Coefficients per 100 people. T-statistic in parenthesis.

Finally, I explore the relationship between car and public transit mode shares. When commuters change the car mode the have different alternatives to chose from. However, I find that the relationship between car and public transit modes is negative 1-to-1. That is, public transit share increases car share decreases by the same amount. Figure (3.3) shows the empirical relationship between public transit shares and car shares at the County level (panel a) and at the US Census tract level (panel b).

Figure 3.3: Car Share - Bus Share



(a) County Level



(b) Census Tract Level

Therefore, the two main takes out of this exploratory exercise are: 1) locations where population density is higher, workers are less likely to chose car as their commuting mode, and 2) the relationship between car and public transit mode shares is 1-to-1 negative correlated. That is, commuters switch from one to the other but not to other mode choices.

3.3 Model

In this section I extend Ahlfeldt et al. (2015) ([**?**])model of internal city structure to allow consumers to chose transportation mode and travel costs to arise endogenously generating this link between population density and mode choice. For simplicity I abstract from residential and employment spillovers.

3.3.1 First Stage: A Model of Internal City Structure

Environment

The city is populated by an endogenous measure of workers H, who are perfectly mobile between the city and the wider economy. The reservation utility in the wider economy is $\bar{U}$. In a first stage, if a worker decides to move to the city, she observes the realization of idiosyncratic taste shocks for any possible pair of residence-employment locations and chooses the one that maximizes her utility.

Locations are heterogeneous in terms of residential amenities (B_i), productivity (A_j) and access to the transportation network ($\mathcal{G} = \{\mathcal{N}, \mathcal{A}\}$).

In a second stage, consumers observe the realization of an idiosyncratic commuting mode test shock and chose transportation mode and route to connect their residence location to their employment location.

Preferences and Demand for Travel

Worker $o \in H$ with residence location i and employment location j derives utility from residential amenities B_i, consumption c_{ijo} (the numeraire, i.e. $p = 1$), floor space l_{ijo} and an idiosyncratic taste shock z_{ijo}. Utility is given by:

$$U_{ijo} = \frac{B_i z_{ijo}}{\rho_{ijo}} \left(\frac{c_{ijo}}{\beta}\right)^{\beta} \left(\frac{l_{ijo}}{1-\beta}\right)^{1-\beta} \tag{3.2}$$

ρ_{ijo} is a commuting cost defined as $\rho_{ij} = e^{\kappa \tau_{ijo}} \in [1, \infty)$, where κ controls the size of the effect of travel cost, τ_{ijo}, in the utility function. At this stage, workers take travel costs as given when making their choices and in a second stage those costs are revealed as an equilibrium in the commuting market.

Idiosyncratic taste shocks, z_{ijo}, are drawn from a Fréchet distribution with scale parameter $T_i E_j$ where $T_i > 0$ and $E_j > 0$ determine the average utility of living in location i and working in location j respectively, and shape parameter $\epsilon > 1$ that controls the dispersion of idiosyncratic utility. The CDF of z_{ijo} is then:

$$F(z_{ijo}) = e^{-T_i E_j z_{ijo}^{-\epsilon}} \quad T_i, E_j > 0, \epsilon > 1 \tag{3.3}$$

With all that, solutions for consumption and land demands for a worker with residence in location i and employment in j are[3]

$$c_{ijo} = \beta w_j \tag{3.4}$$

$$l_{ijo} = (1-\beta)\frac{w_j}{Q_i} \tag{3.5}$$

where w_j is the wage received by working in location j and Q_i is the land price in location i. Plugging them into the utility function (3.2) get the indirect utility function:

$$U = \frac{B_i z_{ijo} w_j Q_i^{\beta-1}}{\rho_{ij}} \tag{3.6}$$

Since the idiosyncratic taste shock enters the utility function (3.2) multiplicatively, the distribution of utility derived from i and j is also Fréchet. A worker chooses locations to maximize utility and since the maximum of a sequence of Fréchet random variables is itself Fréchet, the distribution of utility across all possible pairs of blocks of residence and employment is:

$$1 - G(u) = 1 - \Pi_{r=1}^{S}\Pi_{s=1}^{S} e^{-\Phi_{rs}u^{-\epsilon}} \tag{3.7}$$

This allows us to compute the probability that a worker chooses the pair i, j out of all possible pairs within the city:

$$\pi_{ij} = \frac{T_i E_j \left(\rho_{ij} Q_i^{1-\beta}\right)^{-\epsilon} \left(B_i w_j\right)^{\epsilon}}{\sum_{r=1}^{S}\sum_{s=1}^{S} T_r E_s \left(\rho_{rs} Q_r^{1-\beta}\right)^{-\epsilon} \left(B_r w_s\right)^{\epsilon}} = \frac{\Phi_{ij}}{\Phi} \tag{3.8}$$

That is, workers sort across city locations taking into account their idiosyncratic taste shock and location characteristics. These choice probabilities can be compactly

[3] The budget constraint in the utility maximization problem of consumer o for a given residence-employment pair (ij) is:

$$c_{ijo} + Q_i l_{ijo} = w_j$$

since workers supply inelastically one unit of time.

expressed in a matrix Π:

$$\Pi = \begin{bmatrix} \pi_{11} & \pi_{12} & \dots & \pi_{1S} \\ \pi_{21} & \pi_{22} & \dots & \pi_{2S} \\ \vdots & \vdots & \ddots & \vdots \\ \pi_{S1} & \pi_{S2} & \dots & \pi_{SS} \end{bmatrix}$$

Hence, travel demand from i to j is simply the probability of choosing residence-employment pair (i, j) times total city population H, i.e. $d_{ij} = \pi_{ij} H$. In matrix form:

$$M = \Pi H \tag{3.9}$$

which is the travel demand matrix used in section (3.3.2).

Summing across employment locations s get the probability that a worker chooses to live in location i:

$$\pi_{Ri} = \frac{\Phi_i}{\Phi} \tag{3.10}$$

Instead, summing across residence locations r get the probability that a worker chooses to work at location j:

$$\pi_{Mi} = \frac{\Phi_j}{\Phi} \tag{3.11}$$

Conditional probability of commuting from location i given employment in location j is:

$$\pi_{ij|j} = \frac{T_i E_j \left(\rho_{ij} Q_i^{1-\beta}\right)^{-\epsilon} \left(B_i w_j\right)^{\epsilon}}{\sum_{r=1}^{S} T_r E_j \left(\rho_{rj} Q_r^{1-\beta}\right)^{-\epsilon} \left(B_r w_j\right)^{\epsilon}} = \frac{\Phi_{ij}}{\Phi_j} \tag{3.12}$$

and the conditional probability that a worker commutes to block j conditional on having chosen to live in block i is:

$$\pi_{ij|i} = \frac{E_j \left(w_j / \rho_{ij}\right)^{\epsilon}}{\sum_{s=1}^{S} E_s \left(w_s / \rho_{is}\right)^{\epsilon}} = \frac{\Phi_{ij}}{\Phi_i} \tag{3.13}$$

Commuting market clearing requires that the measure of workers employed in each location j (H_{Mj}) equals the sum across all locations i of their measures of residents

(H_{Ri}) times their conditional probabilities of commuting to j ($\pi_{ij|i}$):

$$\begin{aligned} H_{Mj} &= \sum_{i=1}^{S} \pi_{ij|i} H_{Ri} \\ &= \sum_{i=1}^{S} \frac{E_j \big(w_j/\rho_{ij}\big)^{\epsilon}}{\sum_{s=1}^{S} E_s \big(w_s/\rho_{is}\big)^{\epsilon}} H_{Ri} \end{aligned} \tag{3.14}$$

Expected worker income conditional on living in location i is just the sum of all possible wages in all the locations weighted by the probability of commuting to those locations conditional on living in i.

$$\begin{aligned} \mathbb{E}[w_s|i] &= \sum_{s=1}^{S} \pi_{is|i} w_s \\ &= \sum_{s=1}^{S} \frac{E_s \big(w_s/\rho_{is}\big)^{\epsilon}}{\sum_{r=1}^{S} E_r \big(w_r/\rho_{ir}\big)^{\epsilon}} H_{Ri} \end{aligned} \tag{3.15}$$

where $\mathbb{E}[]$ denotes the expectation operator and the expectation is taken over the distribution of the idiosyncratic component of utility.

Finally, perfect worker mobility between the city and the larger economy implies that worker's expected utility of living in the city is equal to the reservation utility in the wider economy:

$$\gamma \Big[\sum_{r=1}^{S} \sum_{s=1}^{S} T_r E_s (\rho_{rs} Q_r^{1-\beta})^{-\epsilon} (B_r w_s)^{\epsilon} \Big]^{1/\epsilon} = \bar{U} \tag{3.16}$$

where $\gamma = \Gamma(\frac{\epsilon-1}{\epsilon})$ is the gamma function.

Production

Firms produce an homogeneous tradable good under conditions of perfect competition and constant returns to scale. For a given production location j, the problem of the firm is:

$$\max_{H_{Mj}, L_{Mj}} \quad A_j H_{Mj}^{\alpha} L_{Mj}^{1-\alpha} - w_j H_{Mj} - q_j L_{Mj} \tag{3.17}$$

that is, firms choose the amount of labor and commercial floor space to maximize profits. w_j and q_j are the input prices for labor and commercial land respectively and, as explained above, the final good is the numeraire ($p = 1$). Using first order conditions of the maximization problem, the price of commercial floor space is:

$$q_j = (1-\alpha)\Big(\frac{\alpha}{w_j}\Big)^{\alpha/(1-\alpha)} A_j^{1/(1-\alpha)} \tag{3.18}$$

First Stage Equilibrium

In this section I define the equilibrium of the model of internal city structure for a given set of travel times.

Definition Equilibrium: Given model parameters $\{\alpha, \beta, \epsilon, \kappa\}$, exogenous location characteristics $\{T, E, A, B\}$ and reservation utility level in the wider economy $\bar{U}$, an equilibrium for this economy with endogenous travel times is a vector $\{\pi_M, \pi_R, Q, q, w\}$ and total city population H.

For given travel times, τ, the rest of the equilibrium vector and total city populations is the solution of the following set of equations:

Residential and Employment Probabilities

$$\pi_{Ri} = \frac{\sum_{s=1}^{S} T_i E_s (\rho_{is} Q_i^{1-\beta})^{-\epsilon} (B_i w_s)^{\epsilon}}{\sum_{r=1}^{S}\sum_{s=1}^{S} T_r E_s (\rho_{rs} Q_r^{1-\beta})^{-\epsilon} (B_r w_s)^{\epsilon}} \tag{3.19}$$

$$\pi_{Mi} = \frac{\sum_{r=1}^{S} T_r E_i (\rho_{ri} Q_r^{1-\beta})^{-\epsilon} (B_r w_i)^{\epsilon}}{\sum_{r=1}^{S}\sum_{s=1}^{S} T_r E_s (\rho_{rs} Q_r^{1-\beta})^{-\epsilon} (B_r w_s)^{\epsilon}} \tag{3.20}$$

Population Mobility

$$\gamma\Big[\sum_{r=1}^{S}\sum_{s=1}^{S} T_r E_s (\rho_{rs} Q_r^{1-\beta})^{-\epsilon} (B_r w_s)^{\epsilon}\Big]^{1/\epsilon} = \bar{U} \tag{3.21}$$

Profit Maximization

$$q_j = (1-\alpha)\Big(\frac{\alpha}{w_j}\Big)^{\alpha/(1-\alpha)} A_j^{1/(1-\alpha)} \tag{3.22}$$

Residential Land Market Clearing: demand for residential floor space equals the supply of residence floor for each location i

$$(1-\beta)\frac{\mathbb{E}[w_s|i]H_{Ri}}{Q_i} = L_i \tag{3.23}$$

Commercial Land Market Clearing: requires that the demand for commercial floor space equals the supply of commercial floor space.

$$\Big(\frac{(1-\alpha)A_j}{q_j}\Big)^{1/\alpha} H_{Mj} = L_j \tag{3.24}$$

3.3.2 Second Stage: Mode Choice and Traffic Assignment

The second stage of the model takes as given the travel demand matrix (3.9) and performs mode choice and traffic assignment following the model of the first chapter (1). For simplicity I assume that workers are identical except for mode taste shock and that the only travel cost is travel time.

City locations locations are connected through a transportation network $\mathcal{G}$. To link locations i, $j \in S$, workers have available three commuting modes $C = \{walk, bus, car\}$. Let $k \in C$ be a commuting choice. There are different routes connecting i to j for each mode choice. The set R^k_{ij} collects all routes from i to j using mode k. A particular route is $r \in R^k_{ij}$ and is just a collection of links L^r_{ij}. Each link in the network is defined by the time cost necesary to cross it, t_l. Mode-route total travel time is:

$$t^{kr}_{ij} = \sum_{l \in L^r_{ij}} t_l$$

Worker o faced with trip ij has to chose a transportation mode and a route to complete her trip. Her choice set is composed of all mode-route possible combinations connecting i to j, I denote that set $\Omega_{ij} = \cup_{k\in C} R^k_{ij}$.

Before making her choice $kr \in \Omega_{ij}$, the commuter observes all mode-route travel times, t^{jr}_{od} and her mode taste shock, ϵ_{ok}. Her problem is to chose the mode-route combination that minimizes travel costs:

$$\min_{kr \in \Omega_{ij}} \quad \beta^k + \beta t_{ij}^{kr} + \epsilon_{ok} \tag{3.25}$$

Making the standard assumption of Type I Extreme Value errors, individual choice probabilities are given by:

$$p_{ij}^{kr} = \frac{exp\{\beta^k + X_{ij}^{kr}\beta\}}{\sum_{k \in C} exp\{\beta^k + t_{ij}^{kr}\beta\}}$$

Finally, the demand for a mode-route trip (ij) is:

$$m_{ij}^{kr} = s_{ij}^{kr} m_{ij} \tag{3.26}$$

Once we know the mass of commuters using each route we can recover the mass of commuters using each link in the network. To do so, define the following indicator function:

$$\delta_{ijl}^{kr} = \begin{cases} 1 & \text{if } kr \in \Omega_{ij} \text{ uses link } l \\ 0 & \text{otherwise.} \end{cases}$$

This function takes value 1 if mode-route kr uses link l to connect i to j. Summing over all origins, all destinations, and all possible routes we get the mass of commuters using a particular link:

$$m_l = \sum_{i \in S} \sum_{j \in S} \sum_{kr \in \Omega_{ij}} \delta_{ijl}^{kr} m_{ij}^{kr} \tag{3.27}$$

On the supply side of the commuting market there is the city's transportation network. The network takes as input commuting demand and gives back travel costs. Here it is important to distinguish between the road network, that gets congested, and the public transit and street networks that do not.

Let $\mathcal{G} = \{\mathcal{N}, \mathcal{L}\}$ be the city's transportation network. Each $n \in \mathcal{N}$ represents a location in the city or the intersection of links connecting locations. Hence the set of all origins and destinations, S is a subset of all the network's nodes, $S \subset \mathcal{N}$. Links $l \in \mathcal{L}$ represent road, street or rail segments. Each link in the network is defined by it's capacity, c_l, and free-flow travel time, t_l^0. Free-flow travel time is the minimum amount of time required to traverse the link with no other commuter on the link.

The difference between the public (street plus public transit) and the road network is that the latter is congestionable. That is, time and cost depend on how many commuters are on the network and their distribution. To introduce this characteristic let me distinguish between the road network, $\mathcal{G}^R$, and the public network, $\mathcal{G}^P$.

Links in the road network have attached a congestion function relating link usage, m_l, to travel time:

$$t_l = t_j(m_l|c_l, t_l^0)$$

This function is nonnegative, single-valued, monotonically increasing and strictly convex. For a more thorough discussion of these functions and how to estimate their parameters see the second chapter of this dissertation (2).

Equation (3.27) gives the flow of commuters for every link and route costs can be recovered as the sum of link costs. Hence, travel time and cost of route r connecting i to j using route r are:

$$t_{ij}^{kr} = \sum_{l \in L_{ij}^r} t_k(m_l|c_l, t_l^0) \tag{3.28}$$

On the other hand, routes on the public network $\mathcal{G}^P$ do not get congested and hence, trip characteristics are independent of network usage. Hence, for non-car modes ($k \in \{walk, bus\}$) travel time is:

$$t_{ij}^{kr} = \sum_{l \in L_{ij}^r} t_l^0$$

Finally, the city's commuting market is in equilibrium when for a given set of commuting flows, travel times are such that workers chose the routes that give rise to the same set of traffic flows. Moreover, the equilibrium requires that markets clear, that is, the travel demand for each origin-destination pair is equal to the sum of travel flows across transportation modes and routes.

Commuting Market Equilibrium Given travel demand, M, an equilibrium is a set of mode-route worker flows, $\{m^{kr*}\}_{kr\in\Omega_{ij}}\ \forall i,j \in S$, travel times $\{t^{kr*}\}_{kr\in\Omega_{ij}} \forall i,j \in S$, such that:

- Given travel times $\{t^{kr*}\}_{kr\in\Omega_{ij}} \forall i,j \in S$ equilibrium mode-route workers are $\{m^{kr*}(t^*)\}_{kr\in\Omega_{ij}}\ \forall i,j \in S$.
- Given equilibrium mode-route mass of workers $\{m^{kr*}\}_{kr\in\Omega_{ij}}\ \forall i,j \in S$, travel times are $\{t^{kr*}(m^*)\}_{kr\in\Omega_{ij}} \forall i,j \in S$.
- Market clearing: $m_{ij} = \sum_{kr\in\Omega_{ij}} m_{ij}^{kr} \quad \forall i,j \in S$.

Finally, the first stage of the model takes as input a unique matrix of travel times, τ, and the solution to the second stage gives as output three different travel times matrices and mode travel shares. To overcome this problem, I compute the expected travel time to complete trip ij as:

$$\tau_{ij} = \frac{1}{m_{ij}} \sum_{kr\in\Omega_{ij}} m_{ij}^{kr} t_{ij}^{kr} \quad \forall i,j \in S \tag{3.29}$$

3.3.3 Numerical Strategy

Given a matrix of travel times, τ, the system of equations (3.19)-(3.24) gives part of the equilibrium vector, $\{\pi_M, \pi_R, Q, q, w,\}$ and total city population H. To determine the entire equilibrium, including travel times, the following strategy is proposed: for any arbitrary travel time matrix, find the equilibrium objects using the above system of equations, with this equilibrium compute a new travel time matrix that satisfies the definition in (3.3.2) and compare the outcome with the initial travel time matrix[4]. If travel times are different, use the last one to compute a new equilibrium. This is summarized in the following algorithm:

[4]For a more detailed explanation on how to solve the second stage equilibrium see chapter one (1)

Algorithm 3 Equilibrium Finding Algorithm

0: Take as given parameters $\{\epsilon, \kappa, \beta, \alpha\}$, location characteristics $\{T, E, B, A\}$ and network $\mathcal{G}$ with link performance functions and their respective parameters $\{t_l^0, c_l\}$.
0: set $v = 0$
0: ***Step 0****: Initialization* Set $t_l = t_l^0 \quad \forall l$, compute shortest route path on network $\mathcal{G}$ for each $i, j \in S$ and compute travel time matrix τ^v
0: *loop*:
0: **while** $|error| \geq tol$ **do**
0: ***Step 1****: Obtain Equilibrium for given* τ^v Use the system of equations (3.19)-(3.24) to obtain $\{\pi_M^v, \pi_R^v, Q^v, q^v, w^v,\}$ and H^v
0: ***Step 2****: Obtain Travel Demand* Use equation (3.9) with $\{\tau^v, Q^v, w^v\}$ to obtain matrix travel demand M^v.
0: ***Step 3****: Obtain Travel Times* Use M^v in the traffic assignment problem of section (3.3.2) to obtain τ^{v+1}.
0: ***Step 4****: Convergence Test* Set $error = \frac{\|\tau^{v+1} - \tau^v\|}{\|\tau^v\|}$
0: ***Step 5****: Update* Set $\tau^v = \tau^{v+1}$
=0

3.4 Numerical Simulations

In this section I simulate the second stage of the model to explore how different density levels affect travel times and mode choice in the county of Los Angeles.

The reasons that make Los Angeles county a good case of study are twofold: on the one hand, Los Angeles in one of the metropolitan areas with worst traffic in the US[5]. On the other hand, single-family zoning represents more than 73% of total residential land in the city of Los Angeles and the local government is studying changes in the zoning policies[6].

I simulate the typical weekday 8am commute in the county of Los Angeles for different density levels. I obtain the origin-destination travel demand matrix from data

[5]from Texas A&M Transportation Institute and INRIX 2019 Index

[6]From Bloomberg City Lab: '*In super-populous Southern California, local governments are on the hook for more than 1.34 million new units by 2029. While they have been slower on citywide upzoning, updates such as L.A.'s transit-oriented communities program and the "complete communities" housing plan in San Diego are also steps towards the region's big targets.*'

and rescale it to simulate uniform increases in density, that is, increasing uniformly the population across all locations. Then I recover travel times and mode shares. As expected, the model is able to reproduce the link between increases in density and decreases in car use. Moreover, as seen in section (3.2) the decrease in car use translates almost 1 to 1 to increases in public transit use and not in increases in walk share.

3.4.1 Data

The data to simulate the transportation market in Los Angeles County is composed of two main sources: commute data and transportation network data. I briefly describe those sources. For more information see chapter two (2).

Morning Commute Data

As in the second chapter (2), I define residential locations to be US Census tracts and employment locations ZIP codes in Los Angeles county. To simulate a typical 8am morning commute I use the Longitudinal Employer-Household Dynamics (LEHD) Origin-Destination Employment Statistics (LODES) for 2016 to simulate home tract-to-work zip commute flows.

LODES data reports total flow of workers on a typical day from census block to census block. This flows reported are total day flows and are not disaggregated by time blocks. I use the American Community Survey (ACS) departures by 30 minute time bracket estimates to obtain flows by departure time.

Formally, let M^{total} be an origin-destination matrix of commuting flows coming from LODES data, and s_{8am} be a column vector where entries are the share of departures by origin at 8:00am from the ACS. Hence, the benchmark commuting origin-destination demand matrix used in the simulation is:

$$M = s^{8am} \otimes M^{total} = \begin{bmatrix} s_1^{8am} M_{11}^{total} & s_1^{8am} M_{12}^{total} & s_1^{8am} M_{13}^{total} & \dots \\ \vdots & \vdots & \vdots & \dots \end{bmatrix}$$

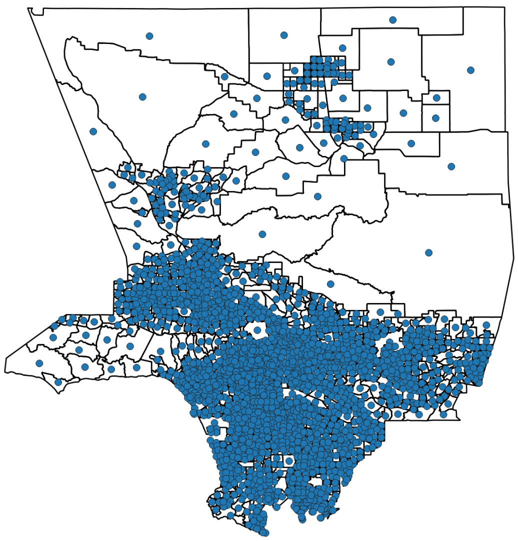
To increase density uniformly across all pair of residential-employment locations I scale this matrix by $\kappa \in \{1, 1.05, 1.1, 1.15, 1.2, 1.25\}$. That is, 5% increments in density up to 25%. That is, the travel demand matrix used as input in the simulation is: $\kappa \times M$.

Transportation Network Data

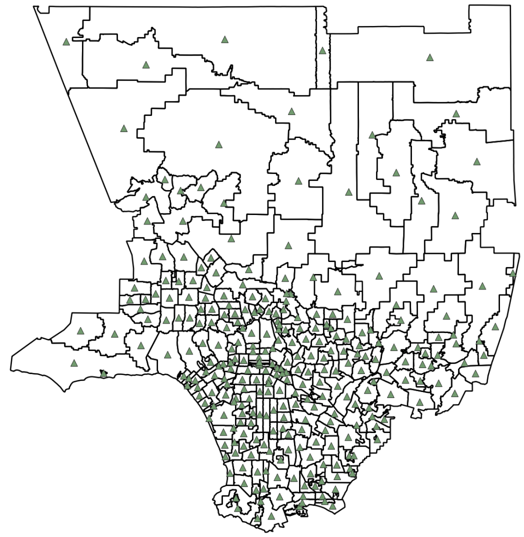
The road network data comes from the Southern California Association of Governments (SCAG). The final network is composed of more than 30.000 links representing segments of roads[7]. I add additional links connecting location centroids to the network. For each link I observe road type, length, posted speed, and number of lanes. With this information I can obtain link capacity and free flow travel time for all links.

Finally, the network consists of 1.623 centroids, 1.443 census tracts that are origin locations and 180 zip codes that are destination locations (see figure (3.4)). Hence there are 1.623 connector links from centroids to the road network. The road network has 29.347 links and 16.247 nodes. See the final road network in figure (3.5).

[7] The original network covers Southern California minus the county of San Diego and is composed of more than 100,000 links. I trim the original network to get only the Los Angeles County network. I further reduce the network by means of an iterative procedure: I perform car traffic assignment and save all the links that get some traffic. I remove these links from the network, making sure that the networks is still fully connected, and perform car traffic assignment again. I delete all links with no flow after these two iterations. I have to do this due to the computational burden that imposes working with the entire network.



(a) Residential Locations



(b) Working Locations

Figure 3.4: Residential and Working Centroids Location

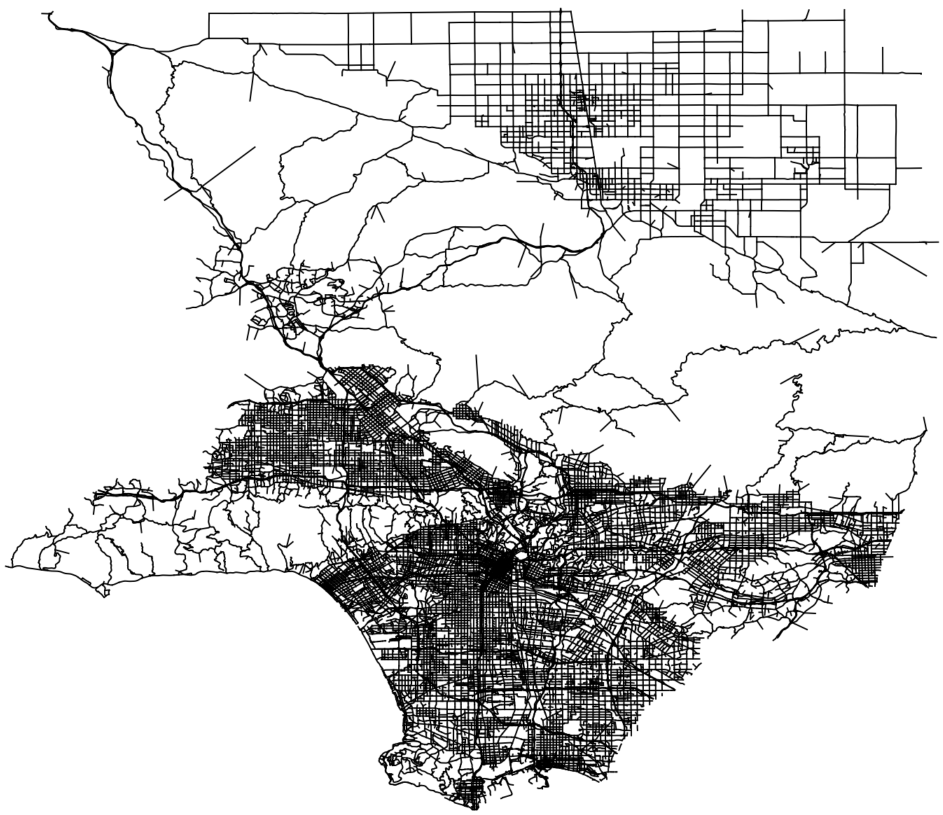


Figure 3.5: Los Angeles County Road Network

To simulate the public transit and street network I use Open Trip Planner (OTP) together with the most updated General Transit Feed Specification (GTFS) Los Angeles Metro schedule data. Open Trip Planner provides directions similar to Google Maps and trips can be planned around an arbitrary public transit schedule. The Los Angeles Metro system is bimodal and consists of a rail and a bus network. The rail network is composed of 2 subway lines, 4 light rail lines, and 93 stations connected by 97.6 miles of rails (see figure (3.6))[8]. On the other hand, the bus system consists of 140 lines, 13,978 bus stations covering 1,433 road miles. The average weekday ridership of the bus system in 2016 is of 1,024,267 passengers[9].

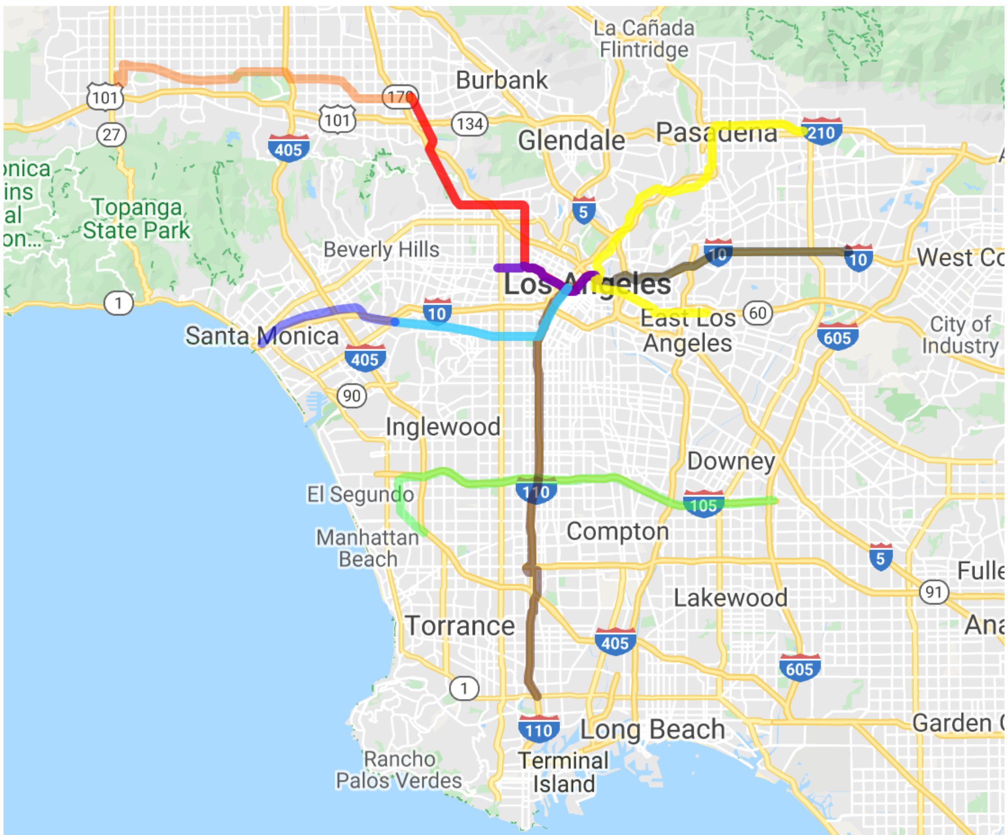


Figure 3.6: Los Angeles County Light Rail Network

[8]Los Angeles Metro Facts at a Glance
[9]Metro LAestimated ridership stats

3.4.2 Simulation and Results

With this data and parameter estimates for the utility function and the congestion function from the second chapter (2) I simulate the second stage of model for different values of κ[10]. An important assumption here is that public transit works by schedule and that congestion in the road network does not affect public transit. Therefore, the more density and more road congestion the same bus schedule. Metro LA has an on-time accuracy of 99.5% in the rail system and of 83.5% on the bus system[11]. We can think of the city allowing for more dedicated lines or adding more buses to ensure that the schedule is met. Tables (3.4) and (3.5) show the results of this exercise.

First, table (3.4) shows changes in average travel times per commuting mode as population density increases. The average car travel time steadily increases from 27.44 minutes up to 29.56 minutes (plus 2.12 minutes and 7.73% increase) for a 25% increase in population density. Public transit and walk average travel times increase relatively less than car travel times: 0.98 minutes and 1.62%, and 0.46 minutes and 1.38% respectively.

[10]For computational details see the first chapter 1).

[11]See here for more information.

Table 3.4: Travel Times - Density, Los Angeles County

	Benchmark (1)	(2)	(3)	(4)	(5)	(6)
Travel Demand	983,762	1,032,950	1,082,138	1,131,326	1,180,514	1,229,702
Change	.	5%	10%	15%	20%	25%
Mean Car t.t.	27.44	28.06	28.44	28.81	29.19	29.56
Change	.	0.62	0.38	0.37	0.38	0.37
Mean Public t.t.	60.6	60.88	61.05	61.21	61.39	61.58
Change	.	0.28	0.17	0.16	0.18	0.19
Mean Walk t.t.	33.22	33.35	33.43	33.51	33.60	33.68
Change	.	0.13	0.08	0.08	0.09	0.08

Next, table (3.5) shows how these changes in travel times translate in changes in commuting mode choices. Car mode share decreases from 77.95% to 76.91% (-1.04 percentage points) for a 25% increase in population density. Public transit follows the opposite direction and increases from 17.23% to 18.17% (+0.94 percentage point)for a 25% increase in population density, and walking share stays constant with a 0.1 percentage points increase.

Note that the model is able to reproduce the patterns in the data observed in section (3.2). A 20% increase in density translated into a change in car share from -0.95 to -0.7 percentage points. The model predicts a -0.82 decrease in car share, which lies in the interval. Moreover, this decrease in car share is absorbed in a 90.3% by the public transit system as seen in the reduced form exercise.

Table 3.5: Mode Share - Density, Los Angeles County

	Benchmark (1)	(2)	(3)	(4)	(5)	(6)
Travel Demand	983,762	1,032,950	1,082,138	1,131,326	1,180,514	1,229,702
Change	.	5%	10%	15%	20%	25%
Car Share	77.95	77.76	77.55	77.35	77.13	76.91
Change	.	-0.19	-0.21	-0.2	-0.22	-0.22
Bus Share	17.23	17.40	17.59	17.77	17.97	18.17
Change	.	0.17	0.19	0.18	0.20	0.20
Walk Share	4.82	4.84	4.86	4.88	4.9	4.92
Change	.	0.02	0.02	0.02	0.02	0.02
Welfare (p.p.)	-1.82	-1.83	-1.84	-1.85	-1.87	-1.88
Change (p.p.)	.	-0.01	-0.01	-0.01	-0.02	-0.01

The logic of the model is as follows: initially, an increases in population density creates increases in car travel times due to the presence of more cars in the road network that generates increases in traffic congestion. These increases in car travel times make commuting by public transit and walking more attractive options. Trips that where previously performed by car are now done using public transit or walking even if these modes' travel times are a longer. As a result, average car mode share decreases and average public transit and walking shares increase even if travel times in in all three commuting modes increase.

3.5 Final Remarks

The recent public debate on single-family zoning laws has the potential to free a sizable amount of urban land for development and create denser metropolitan areas.

Increases in population density, among others, will have an impact on traffic congestion and the commuting choices of citizens.

In this chapter I have documented the link between population density and commuting mode choice. I start by showing that 20% increases in population density are correlated with -0.95 to -0.7 percentage point decreases in car mode share. Furthermore, changes in car travel mode are absorbed, almost 1-to-1 by the public transit system.

Next, I propose a model of internal city structure where people want to exploit density benefits in the form of residential and production amenities but this generates traffic congestion that translates into travel costs. At their time, increases in travel costs will affect commuters mode choices.
The main difference with the existing models of internal city structure as Ahlfeldt et. al. (2015) ([**?**]) is that my model can map changes in population density into choices on the commuting market. This is achieved by expanding their model to allow commuters to make mode and routing choices and where travel costs arise endogenously as a function of the transportation network use.

I then run a simulation of the second stage of the model in Los Angeles County to see how different density levels, where density is increased uniformly across all the city, impact travel times and commuting mode shares. The model captures the two main characteristics highlighted before: a 20% increase in population density translates into -0.82 percentage points in car mode share and 90.3% of this change is absorbed by the public transit system.

The last part remaining is to exploit the whole structure of the model to simulate different changes in zoning policies. The first stage of the model will determinate how population density changes across different city locations. An then, the second stage of the model will translate these changes in population density into changes in mode choices. However, this is left as future work.

3.6 Appendix

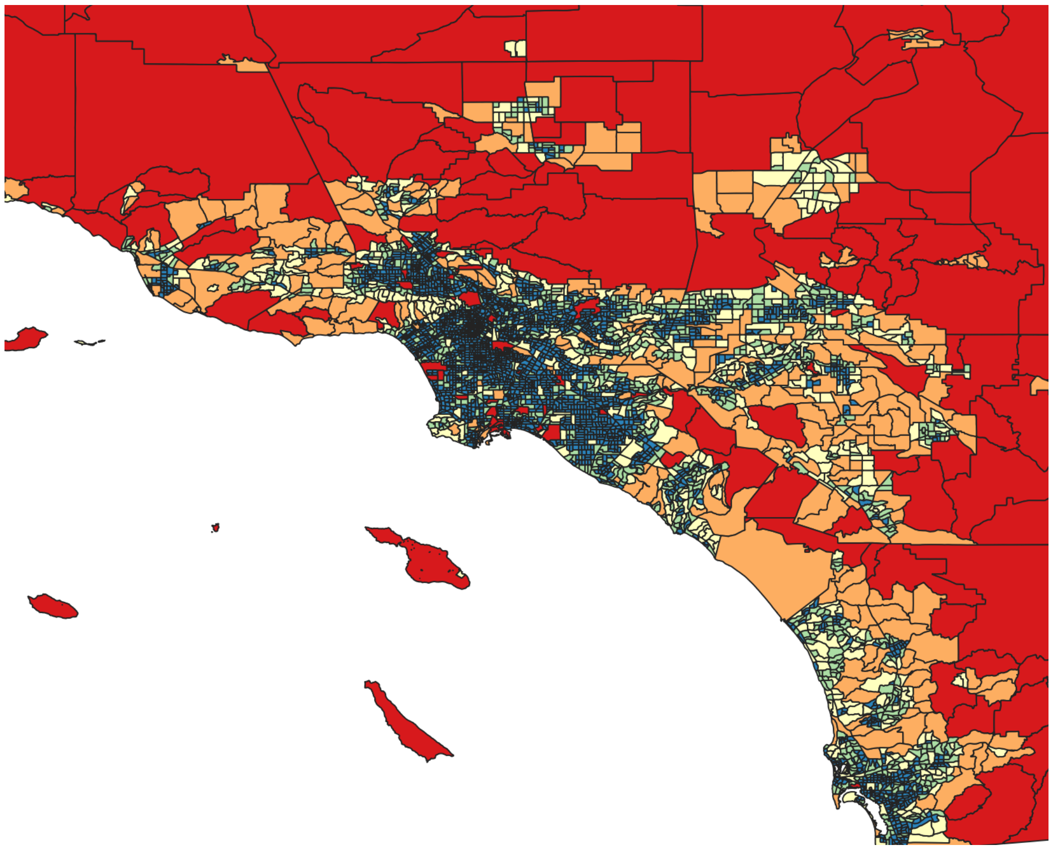


Figure 3.7: Population Density by Census Tract: Los Angeles - San Diego

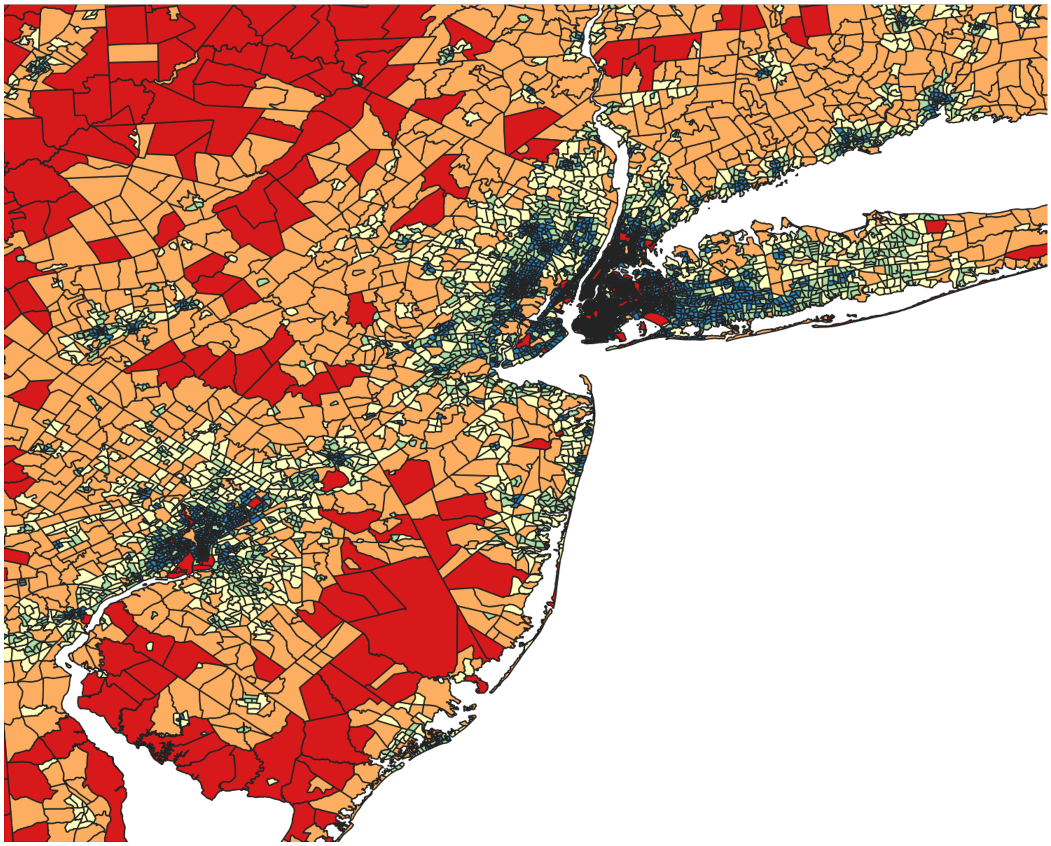


Figure 3.8: Population Density by Census Tract: New York City - Philadelphia

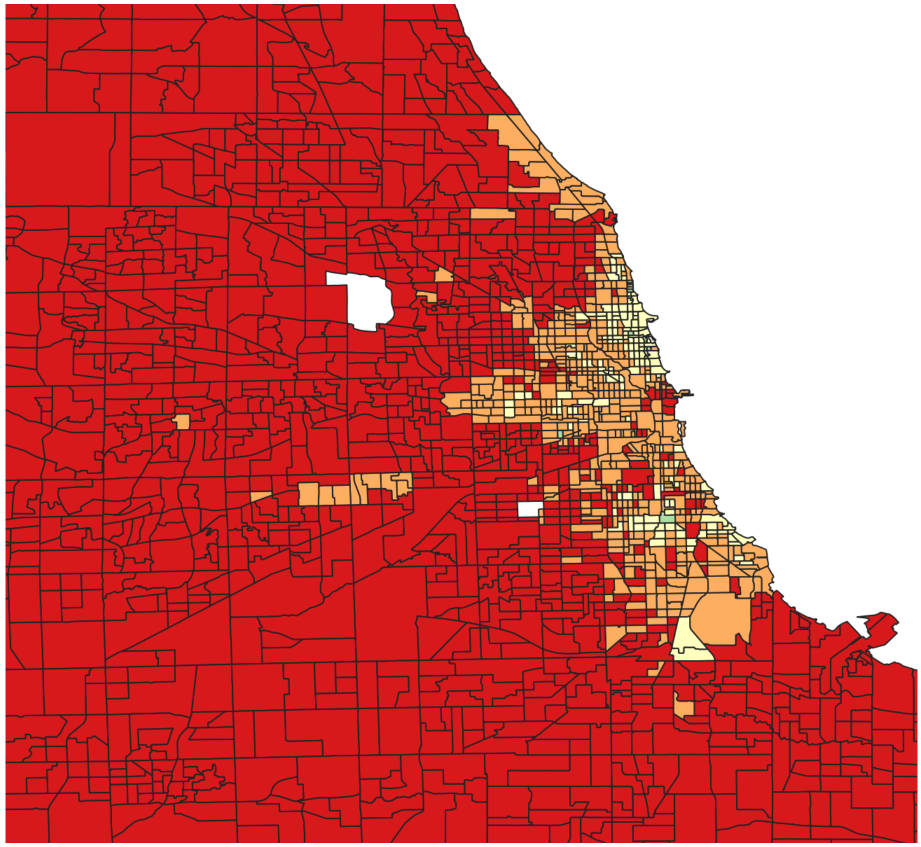


Figure 3.9: Public Transit Share by Census Tract: Chicago

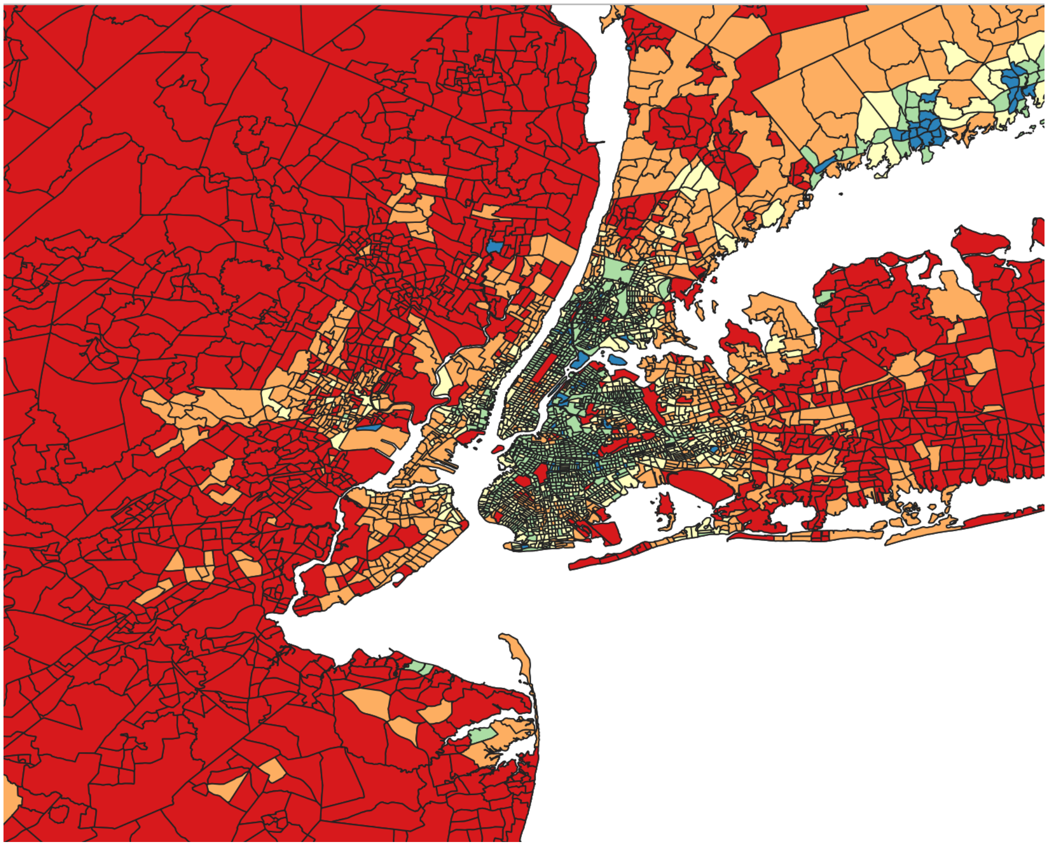


Figure 3.10: Public Transit Share by Census Tract: New York City - Philadelphia

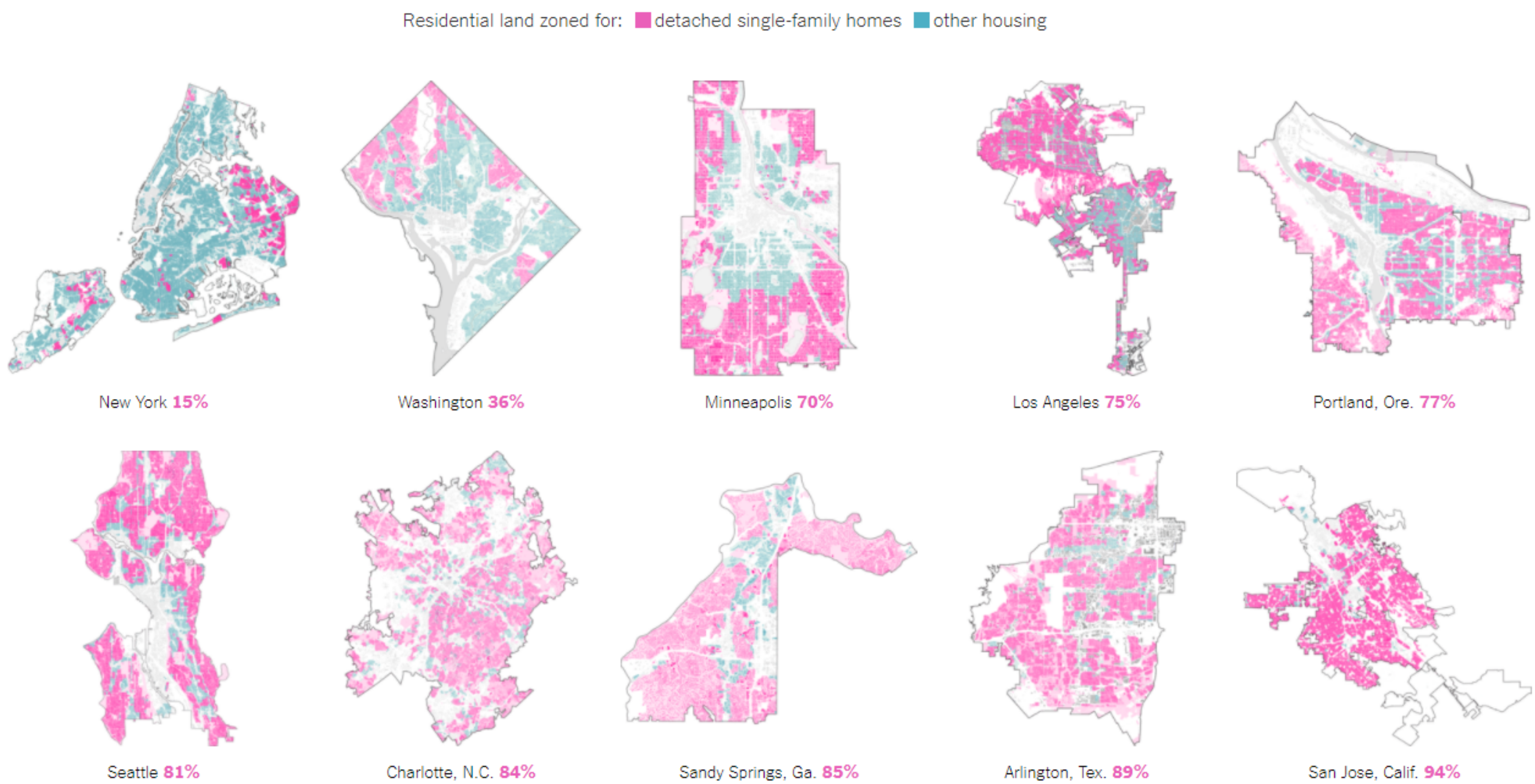


Figure 3.11: Singe-family zoning in US cities
Cities not to scale. Source: *Cities Start to Question an American Ideal: A House With a Yard on Every Lot*, the New York Times, June 2019.

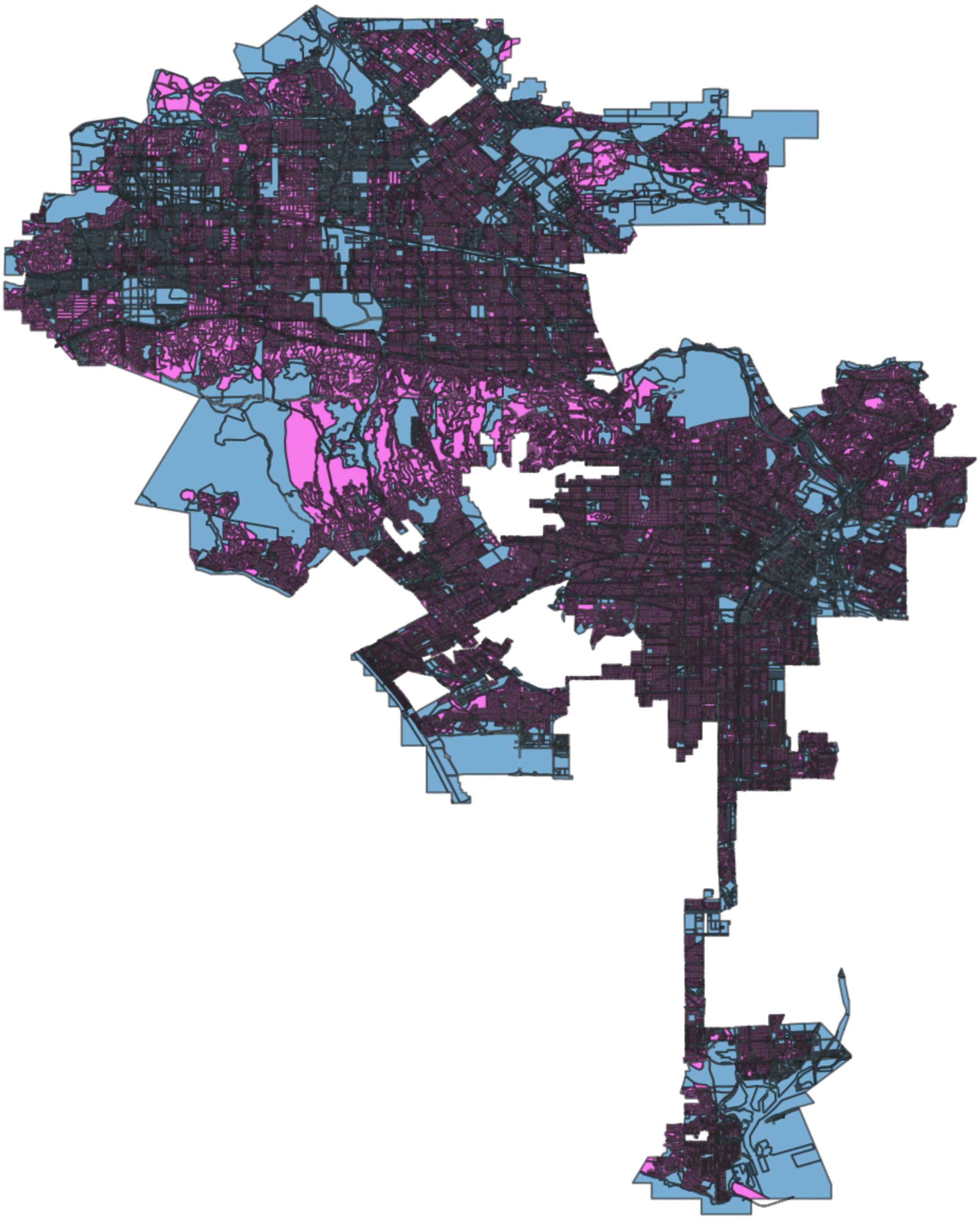


Figure 3.12: Detail of single-family zoning in the city of Los Angeles
Single-family zones in pink, all other zones in blue. Source: the Los Angeles Planning and Zoning Municipal Code and the Generalized Summary of Zoning Regulations, City of Los Angeles

Bibliography

[1] Gabriel M. Ahlfeldt, Stephen J. Redding, Daniel M. Sturm, and Nikolaus Wolf. The economics of density: Evidence from the berlin wall. *Econometrica*, 83(6):2127–2189, 2015.

[2]

[3] Michael L. Anderson. Subways, strikes, and slowdowns: The impacts of public transit on traffic congestion. *American Economic Review*, 104(9):2763–96, September 2014.

[4] William S. Vickrey. Pricing in urban and suburban transport. *The American Economic Review*, 53(2):452–465, 1963.

[5] Richard Arnott, André de Palma, and Robin Lindsey. A structural model of peak-period congestion: A traffic bottleneck with elastic demand. *The American Economic Review*, 83(1):161–179, 1993.

[6] Gilles Duranton and Matthew A. Turner. The fundamental law of road congestion: Evidence from us cities. *The American Economic Review*, 101(6):2616–2652, 2011.

[7] Victor Couture, Gilles Duranton, and Matthew A. Turner. Speed. *The Review of Economics and Statistics*, 0(0):null, 2016.

[8] Prottoy Akbar and Gilles Duranton. Measuring the cost of congestion in highly congested city: Bogota. *CAF – Working paper: Num 2017 04*, 2017.

[9] Gabriel M. Ahlfeldt, Stephen J. Redding, and Daniel M. Sturm. The qantitative evaluation of urban transport infrastructure improvements. 2016.

[10] Treb Allen and Costas Arkolakis. The welfare effects of transportation infrastructure improvements. 2016.

[11] Nick Tsivanidis. Evaluating the impact of urban transit infrastructure: Evidence from bogota's transmilenio. *Working Paper*, 2019.

[12] Yosef Sheffi. Urban transportation networks: Equilibrium analysis with mathematical programming methods. 1984.

[13] M Florian and D W Hearn. Traffic assignment: Equilibrium models. *Proceedings of the Institution of Mechanical Engineers, Part I: Journal of Systems and Control Engineering*, 215(4):305–315, 2001.

[14] José R. Correa and Nicolás E. Stier-Moses. Wardrop equilibria. 2011.

[15] M.J. Beckmann, C.B. McGuire, and C.B. Winston. Studies in the economics of transportation. 1956.

[16] Rafa Kucharski and Arkadiusz Drabicki. Estimating macroscopic volume delay functions with the traffic density derived from measured speeds and flows. *Journal of Advanced Transportation*, 2017.

[17] Erick Guerra, Robert Cervero, and Daniel Tischler. Half-mile circle: Does it best represent transit station catchments? *Transportation Research Record*, 2276(1):101–109, 2012.

[18] Pedro Tanuere Veloso. Housing supply constraints and the distribution of economic activity: The case of the twin cities. *Working Paper*, 2021.

[19] Treb Allen and Costas Arkolakis. Trade and the topography of the spatial economy. *The Quarterly Journal of Economics*, 129(3):1085–1140, 2014.

[20] Treb Allen, Costas Arkolakis, and Xiangliang Li. Optimal city structure. 2017.

[21] Ferdinando Monte, Stephen J. Redding, and Esteban Rossi-Hansberg. Commuting, migration and local employment elasticities. *American Economic Review*, forthcoming.

[22] Stephen J. Redding and Esteban Rossi-Hansberg. Quantitative spatial economics. *Annual Review of Economics*, 9(1):21–58, 2017.

[23] Edward Glaeser and Joseph Gyourko. The economic implications of housing supply. *Journal of Economic Perspectives*, 32(1):3–30, February 2018.

[24] Andrii Parkhomenko. Local causes and aggregate implications of land use regulation. 2020.

[25] Prottoy Akbar and Gilles Duranton. Measuring the cost of congestion in a highly congested city: Bogota. 2017.

[26] Prottoy Akbar, , Victor Couture, Gilles Duranton, and Adam Storeygard. Mobility and congestion in urban india. 2018.

[27] Zhiyuan Yao and Changjoo Kim. The changes of urban structure and commuting: An application to metropolitan statistical areas in the united states. *International Regional Science Review*, 42(1):3–30, 2019.

[28] Zhenjun Zhu, Zhigang Li, Ye Liu, Hongsheng Chen, and Jun Zeng. The impact of urban characteristics and residents' income on commuting in china. *Transportation Research Part D: Transport and Environment*, 57:474–483, 2017.